On the Model Selection Properties and Geometry of the Lasso

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We investigate the model selection properties of the Lasso estimator in finite samples with absolutely no conditions on the regressor matrix $X$. We show that which covariates the Lasso estimator may potentially choose in high dimensions (where the number of explanatory variables $p$ exceeds sample size $n$) depends only on $X$ and the given penalization weights. This set of potential covariates can be determined through a geometric condition on $X$ and may be small enough (less than or equal to $n$ in cardinality) so that the Lasso estimator acts as a low-dimensional procedure also in high dimensions. Related to the geometric conditions in our considerations, we also provide a necessary and sufficient condition for uniqueness of the Lasso solutions.