A DSGE Model of Southern European Capitalism and the Great Recession
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I INTRODUCTION

II MODEL
   II.1 Economic agents and their behavior
   II.2 Private Sector Equilibrium (PSE)
   II.3 PSE Steady State

III UNDERSTANDING THE MODEL
   III.1 Effects and Wedges
   III.2 Sensitivity Analysis of PSE Steady State
   III.3 PSE IRFs

IV POLITICOECONOMIC EQUILIBRIUM (PE)
   IV.1 Government’s Objective Function and the Implementability Constraint
   IV.2 PE Asymptotic Steady State

   IV.3 Sensitivity Analysis of PE Steady State

V INSIDERS INFLUENCE ON GOVERNMENT IN SOUTHERN EUROPEAN COUNTRIES AND ITS EFFECT ON THE ECONOMY’S PROPAGATION MECHANISM
   VI.1 Bayesian Estimation
   VI.2 PE IRFs

VI CONCLUSION (Southern European economies suffered more from the Great Recession than other countries because their politicoeconomic system is characterized by an “insiders – outsiders society” structure.)
I. INTRODUCTION

Source: OECD, National Accounts, Per Capita GDP (Constant prices, constant PPP, OECD base year), 1995-2016
MAIN IDEA: SOUTH EUROPEAN COUNTRIES POLITICOECONOMIC SYSTEM IS AN INSIDERS - OUTSIDERS SOCIETY (Kollintzas, Papageorgiou and Vassilatos ((CEPR, 2012, EJPE, 2018a, IZA Labor Policy 2018b), Kollintzas and Pechlivanos (work in progress, 2018))


NEW IN THIS PAPER: (1) DSGE incorporating the politicoeconomic system of an insiders – outsiders society that leads to a new shock propagation mechanism: Government has an incentive to raise the share of insiders’ industries in the economy in response to a negative shock makes recessions deeper and longer. (2) US is the control group characterized by the corresponding two sector RBC model.
II. MODEL

II.1 Economic Agents and their Behavior

1. Final Good Producers (Cole and Ohanian (JPE, 2004))

Production technology:

\[ y_t = A_t \left( \left( Y_t^i \right)^{\phi} + \left( Y_t^o \right)^{\phi} \right)^{\frac{1}{(1-\phi)}} ; t \in \mathbb{N}_+, \phi < 1 \]  \hspace{1cm} (I.1)

- \( y_t \): output of representative final good producer in period \( t \)
- \( Y_t^i \): aggregate input of intermediate goods of insiders’ sector
- \( Y_t^o \): aggregate input of intermediate goods of outsiders’ sector
- \( A_t \): (static) total factor productivity
- \( \frac{1}{1-\phi} \): input elasticity of substitution across sectors

\[ Y_t^i = \left[ \int_0^{\gamma} y_i(\zeta)^{\theta} d\zeta \right]^{\frac{1}{1-\theta}} ; \theta \in (0,1) \]  \hspace{1cm} (I.2)

\[ Y_t^o = \left[ \int_{\gamma}^{\infty} y_i(\zeta)^{\theta} d\zeta \right]^{\frac{1}{1-\theta}} \]  \hspace{1cm} (I.3)
\( \chi_i \): fraction of insiders’ industries in the economy

\( y_i^i(\varsigma) \): input of the intermediate good of the \( \varsigma \) industry in insiders’ sector

\( y_i^o(\varsigma) \): input of the intermediate good of the \( \varsigma \) industry in outsiders’ sector

\( \frac{1}{\eta} \): aggregation elasticity of substitution across industries

**Profits:**

\[
\pi_i^r = \int_0^{\chi_i} p_i^i(\varsigma) y_i^i(\varsigma) d\varsigma - \int_{\chi_i}^1 p_i^o(\varsigma) y_i^o(\varsigma) d\varsigma
\]

\( p_i^i(\varsigma) \): real price of the intermediate good of the \( \varsigma \) industry in the insiders’ sector

\( y_i^o(\varsigma) \): real price of the intermediate good of the \( \varsigma \) industry in the outsiders’ sector
Behavior:

\[ p_i^i(\varepsilon) = \Gamma_i^i y_i(\varepsilon)^{\theta-1} \] (I.5)

\[ p_i^o(\varepsilon) = \Gamma_i^o y_i^o(\varepsilon)^{\theta-1} \] (I.6)

where:

\[ \Gamma_i^i \equiv A_i^i y_i^{(1-\phi)} y_i^{\phi-\theta} \] (I.7)

\[ \Gamma_i^o \equiv A_i^o y_i^{(1-\phi)} y_i^{\alpha(\phi-\theta)} \] (I.8)

2. Intermediate Good Producers in the Outsiders’ Sector (Cole and Ohanian (JPE, 2004))

Production technology:

\[ y_i^o(\varepsilon) = B_i^o k_i^o(\varepsilon)^{\alpha} l_i^o(\varepsilon)^{(1-\alpha)}; \alpha \in (0,1) \] (I.9)

\[ k_i^o(\varepsilon) : \text{capital input} \]
\[ l_i^o(\varepsilon) : \text{labor input} \]
\[ B_i^o : \text{total factor productivity in outsiders’ industries} \]
\[ \alpha : \text{capital input elasticity} \]
**Profits:**

\[
\pi_i^o(\varsigma) = p_i^o(\varsigma)y_i^o(\varsigma) - r_i k_i^o(\varsigma) - w_i^o l_i^o(\varsigma)
\]  \hspace{1cm} (I.10)

- \( r_i \): real rental cost of capital
- \( w_i^o \): real wage rate of outsiders

**Technical progress:**

We assume labor augmenting technical progress at a constant growth rate \((1 + \eta)\). Thus, output, capital input and the wage rate are expressed in efficient household units and labor input is expressed in household units.

**Behavior:**

\[
\begin{align*}
    r_i &= \alpha \frac{p_i^o(\varsigma)y_i^o(\varsigma)}{k_i^o(\varsigma)} \\
    w_i^o &= (1 - \alpha) \frac{p_i^o(\varsigma)y_i^o(\varsigma)}{l_i^o(\varsigma)}
\end{align*}
\]  \hspace{1cm} (I.11, I.12)
3. Intermediate Good Producers in the Insiders’ Sector

Production technology:

\[ y_i'(\zeta) = B_i k_i'(\zeta)^{\alpha} l_i'(\zeta)^{(1-\alpha)} \]  \hspace{1cm} (I.13)

- \( B_i' \): total factor productivity in insiders’ industries

Profits:

\[ \pi_i'(\zeta) = p_i'(\zeta) y_i'(\zeta) - r_i k_i'(\zeta) - w_i'(\zeta) l_i'(\zeta) \]  \hspace{1cm} (I.14)

- \( w_i'(\zeta) \): real wage rate in the \( \zeta \) industry of the insiders’ sector
Behavior:

\[
    u'_i(\zeta) = \left[ w'_i(\zeta) - w^o_i \right]^{\lambda} \left[ t'_i(\zeta) \right]^{(1-\lambda)} ; \lambda \in (0,1)
\]  \hspace{2cm} (I.15)

\[\lambda\] : preference intensity of wage premium over union membership/employment

Behavior in each industry of the insiders’ sector is defined as follows:

(i) The producer behaves like a monopolist, setting the output price subject to the (inverse) demand function (I.5) and output and inputs are constrained by the production technology (I.13).

Both the producer and the union in each industry of the insiders’ sector take as given:

(ii) the economy and insiders’ sector aggregates \( y \) and \( y^I \), respectively.
(iii) the real rental cost of capital and the real wage rate in the outsiders’ sector, \( r \) and \( w^o \), respectively.
(iv) the fraction of insiders’ industries in the economy, \( \chi \).
Further, behavior in any given industry $\varsigma$ of the insiders’ sector, is characterized by a two-stage game, whereby:

(v) in the first stage, the producer and the union agree upon a wage – employment Nash bargaining contract,

\[
\left( w^*_i(\varsigma), l^*_i(\varsigma) \right) = \underset{(w_i(\varsigma), l_i(\varsigma))}{\text{arg max}} \left\{ \left[ u^*_i(\varsigma) - \bar{w}_i(\varsigma) \right]^{\mu} \left[ \bar{\pi}_i(\varsigma) - \bar{\pi}_i(\varsigma) \right] \right\}; \mu > 0
\] (I.16)

$\bar{u}_i(\varsigma)$: union’s reservation option
$\bar{\pi}_i(\varsigma)$: producer’s reservation option
$\mu$: union’s relative bargaining power

(vi) in the second stage, the producer chooses capital input, so as to maximize profits associated with the wage rate and employment determined in the first stage:

\[
k^*_i(\varsigma) = \underset{k_i(\varsigma)}{\text{arg max}} \left\{ A^i\gamma_i^{1(1-\theta)}Y_i^{1(1-\theta)} \left\{ B_i [k_i(\varsigma)]^{1(1-\alpha)} \right\} - r_i k_i(\varsigma) - \left[ w^*_i(\varsigma) \right] [l^*_i(\varsigma)] \right\}
\] (I.17)

Finally, we assume that if there is no contract:

(vii) union members can work in the industries of the outsiders’ sector for $w^\circ_i$, so that $\bar{u}_i(\varsigma) = 0$.
(viii) the producer can operate in the industries of the outsiders’ sector, earning $\pi^\circ_i$, so that $\bar{\pi}_i(\varsigma) = 0$. 
Proposition 1: Provided that:

\[ \lambda \mu < \frac{1 - \alpha \theta}{1 - \theta} \]

behavior in any given industry \( \zeta \) of the insiders’ sector is such that:

\[
\begin{align*}
    r_i &= \alpha \theta \frac{P_i'(\zeta) \nu_i'(\zeta)}{k_i'(\zeta)} \\
    w_i'(\zeta) &= (1 - \alpha)\nu \bar{P}_i'(\zeta) \frac{P_i'(\zeta) \nu_i'(\zeta)}{l_i'(\zeta)} \\
    \frac{w_i'(\zeta)}{w_i''} &= \nu \\
    \pi_i'(\zeta) &= (\kappa - 1) w_i'(\zeta) l_i'(\zeta)
\end{align*}
\]

where:

\[
\begin{align*}
    \kappa &= \frac{1 + (1 - \lambda) \mu}{(1 - \alpha \theta) + (1 - \lambda) \mu} > 1 \\
    \nu &= \frac{1}{1 - \lambda \mu (\kappa - 1)} > 1
\end{align*}
\]
\[ \xi = \frac{1 - \alpha \theta}{(1 - \alpha) \kappa V} < 1 \] (I.24)

4. **Households**

**Flow Budget Constraint:**

\[ c_t + (1 + \eta) k_{t-1} \leq [1 + (1 - \tau^K_t)(r_t - \delta)] k_t + (1 - \tau^K_t)[\chi_t w^I_t h^I_t + (1 - \chi_t) w^O_t h^O_t + \chi_t \pi^I_t]; \delta \in (0,1) \] (I.37)

- \( c_t \): (final good) consumption
- \( k_t \): capital stock at the beginning of period \( t \)
- \( \chi_t, h^I_t \): labor supply to the insiders’ sector
- \( (1 - \chi_t) h^O_t \): labor supply to the outsiders’ sector
- \( \chi_t \pi^I_t \): dividends from outsiders’ industries
- \( \tau^K_t \): capital income tax rate
- \( \tau^I_t \): labor income tax rate
- \( \delta \): fixed (geometric) capital depreciation rate

**Time Constraint:**

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13
\( \chi_i h_i^l + (1 - \chi_i) h_i^o = h_i \leq 1 \) \hspace{1cm} (I.38)

- \( h_i \): fraction of available household time devoted to work
- \( 1 - h_i \): fraction of available household time devoted to leisure

**Rationing Constraint:**

\( h_i^l \leq l_i \) \hspace{1cm} (given) \hspace{1cm} (I.39)

**Physical Constraints:**

\( c_i, 1 - h_i, h_i^l, h_i^o, k_{i+1} \geq 0 \) \hspace{1cm} (I.40)
Preferences:

\[ U_0^h = E_0 \sum_{t=0}^{\infty} \beta^t u_t^h \]  \hspace{1cm} (I.41)

\[ u_t^h = \frac{e_t^\gamma (1-h_t)^{-\beta}}{1-\gamma}; \gamma < 1 \]  \hspace{1cm} (I.42)

\( \beta \in (0,1) \) is the efficient household constant discount factor

\( 1/\gamma \) intertemporal elasticity of substitution

\( \gamma \) preference intensity for consumption
Environment and Timeline

The economic environment is characterized by the exogenous state of the economy, \( z_t \). The latter, in turn, is characterized by the vector of the three total factor productivities \((A^t_t, B^t_t, \beta^t)\). All economic agents know \( z_t \) at the beginning of period \( t \). In any given period \( t \), government moves first and chooses the government policy variables, based on the information available at the beginning the period, \( z' = \{z_{t'}\}_{t'=0}^{t} \). That is, \( (g_t, \tau^t_t, \chi_t) = (g_t(z'), \tau^t_t(z'), \tau^t_t(z'), \chi_t(z')) \), where \( g_t \) is the output share of government consumption. Private sector economic agents move second and make their decisions, given \( (g_t, \tau^t_t, \chi_t) \), based on \( z' \). In particular, households choose a contingency plan \( \{c_t(z'), h_t(z'), h_t^0(z'), h_t^\gamma(z'), k_t(z')\}_{t'=0}^{z} \). The probability of any particular history of the exogenous state \( z' \), \( \text{prob}(z') \), is given. \( E_t(\bullet) \) stands for the expectations operator, based on the information available at the beginning of period \( t \), \( z' \). As usual, in DSGE analysis, \( \text{prob}(z') \) will be specified indirectly, via the laws of motion of the exogenous state, later.
Behavior:

\[ \frac{\theta(1 - \tau^L_i)w^L_i}{c_i} = \frac{1 - \theta}{1 - h_i} \]  \quad (1.43)

Euler condition:

\[ \frac{u_t}{c_t} = \frac{\beta}{(1 + \eta)} E_0 \left\{ \frac{u_{t+1}}{c_{t+1}} \left[ 1 + (1 - \tau^k_{t+1})(r_{t+1} - \delta) \right] \right\} \]  \quad (1.44)

Transversality condition:

\[ \beta^T E_0 \frac{u_{r+1}k_{t+1}}{c_r} \to 0 \text{ as } T \to \infty \]  \quad (1.45)
II.2 Private Sector Equilibrium (PSE)

1. **Definition:** Given any sequence of the exogenous state \( \{z_i\}_{i=0}^{\infty} \) and any sequence of government policy variables \( \{g_i(z'), \tau^K_i(z'), \tau^L_i(z'), \tau^\chi_i(z')\}_{i=0}^{\infty} \), a private sector equilibrium is a sequence of the form:

\[ \{y_i(z'), y'_i(z'), y''_i(z'), k_i(z'), k'_i(z'), l'_i(z'), l''_i(z'), c_i(z'), h_i(z'), h'_i(z'), h''_i(z'), p_i(z'), p'_i(z'), p''_i(z'), r_i(z'), w_i(z'), w'_i(z'), \pi_i(z')\}_{i=0}^{\infty} \]

such that: (I.1)-(I.3), (I.5)-(I.6), (I.9), (I.11)-(I.13), (I.18)-(I.21), (I.37)-(I.39), (I.43)-(I.45) are satisfied, along with the market clearing conditions:

\[ k_i = \chi_i k'_i + (1 - \chi_i) k''_i \]  
\[ h'_i = l'_i \]  
\[ h''_i = l''_i \]  
\[ (1 - g_i) y_i = c_i + [(1 + \eta) k_{i+1} - (1 - \delta) k_i] \]
Proposition 2: The private sector equilibrium defined above has the following properties:

(a) Outputs and inputs in the insiders’ and outsiders’ industries can be expressed in terms of the exogenous variables, $z_i$, the government policy variable, $\chi_i$, and the aggregate state of the private sector, $(k_i, h_i)$, as follows:

\[
\frac{\chi_i k_i^i}{(1 - \chi_i)k_i^o} = \theta \Delta_i(\chi_i) 
\]  
(II.5)

\[
(1 - \chi_i)k_i^o = \frac{1}{1 + \theta \Delta_i(\chi_i)} k_i 
\]  
(II.6)

\[
\frac{\chi_i h_i^i}{(1 - \chi_i)h_i^o} = \xi \Delta_i(\chi_i) 
\]  
(II.7)

\[
(1 - \chi_i)h_i^o = \frac{1}{1 + \xi \Delta_i(\chi_i)} h_i 
\]  
(II.8)

\[
\frac{\chi_i y_i^i}{(1 - \chi_i)y_i^o} = \left[ \theta^\alpha \xi^{(1-\alpha)} \left( \frac{B_i^i}{B_i^o} \right) \right] \Delta_i(\chi_i) 
\]  
(II.9)
\[(1 - \chi_t) y_t^o = B_t^o \left[ \frac{1}{1 + \theta \Delta_t(\chi_t)} \right]^\alpha \left[ \frac{1}{1 + \xi \Delta_t(\chi_t)} \right]^{(1-\alpha)} k_t^\alpha h_t^{(1-\alpha)} \]  
(II.10)

where:

\[
\Delta_t(\chi_t) \equiv \left[ \theta^\omega \xi^{(1-\alpha)} \left( \frac{B_t}{B_t^o} \right) \right]^{\gamma_{(1-\alpha)}} \left( \frac{\chi_t}{1 - \chi_t} \right)^{\theta(1-\theta)/(1-\theta)\theta} 
\]  
(II.11)

(b) Factor prices, insiders’ profits, and aggregate output can be expressed in terms of the exogenous variables, \( z_t \), the government policy variable, \( \chi_t \), and the aggregate state of the private sector, \((k_t, h_t)\), as follows:

\[
r_t = \alpha \bar{\omega}_t(\chi_t) \frac{y_t}{k_t} 
\]  
(II.12)

\[
w_t^o = (1 - \alpha) \bar{\omega}_t(\chi_t) \frac{y_t}{h_t} 
\]  
(II.13)

\[
\chi_t \pi_t^1 = \frac{(1 - \alpha \theta)(\kappa - 1)}{(1 - \alpha) \kappa} \Delta_t(\chi_t)(1 - \chi_t) w_t^o h_t^o 
\]  
(II.14)

\[
y_t = A^o B_t^o \bar{\omega}_t(\chi_t) k_t^\alpha h_t^{(1-\alpha)} 
\]  
(II.15)
where:

\[ \tilde{\omega}_i(\chi_i) = \frac{1 + \theta \Delta_i(\chi_i)}{1 + \Delta_i(\chi_i)} \]  

(II.16)

\[ \tilde{\omega}_i(\chi_i) = \frac{1 + \xi \Delta_i(\chi_i)}{1 + \Delta_i(\chi_i)} \]  

(II.17)

\[ \tilde{\omega}_i(\chi_i) \equiv \frac{(1 - \chi_i)^{(1-\phi)/\theta}[1 + \Delta_i(\chi_i)]^{\phi}}{[1 + \theta \Delta_i(\chi_i)]^{\phi}[1 + \xi \Delta_i(\chi_i)]^{\phi}} = \frac{(1 - \chi_i)^{(1-\phi)/\theta}[1 + \Delta_i(\chi_i)]^{(1-\phi)/\theta}}{\tilde{\omega}_i(\chi_i)^{\phi} \tilde{\omega}_i(\chi_i)^{(1-\phi)/\theta}} \]  

(II.18)

(c) Suppose that the government runs a balanced budget, in the sense that:

\[ g, y, = \tau_k^i (\kappa_i - \delta) k_i + \tau_l^i [\chi_i w_i h_i^e + (1 - \chi_i) w_i h_i^e + \chi_i \pi_i] \]  

(II.19)

Then, given \( z_i \) and government policy variables \( (g_i, \tau_k^i, \tau_l^i, \chi_i) \), the laws of motion of the aggregate state of the private sector equilibrium: (i) depend only on the aggregate state of the private sector \( (k_i, h_i) \); and (ii) they are characterized, completely, in terms of the intratemporal condition for the household’s labor-leisure choice (I.43),

21
the Euler condition for capital (I.44), the transversality condition (I.45), the resource constraint (II.4) and the government budget constraint (II.19).
II.3 PSE Steady State

*Great Ratios:*

From the Euler condition for capital (I.44):

\[
\left( \frac{k}{y} \right) = \frac{\alpha \beta (1 - \tau^k) \omega(\chi)}{(1 + \eta) - \beta [1 - (1 - \tau^k) \delta]}
\]  
(II.23)

From the resource constraint (II.4):

\[
\left( \frac{c}{y} \right) = (1 - g) - (\eta + \delta) \left( \frac{k}{y} \right)
\]
(II.24)

From the household intratemporal condition (I.43):

\[
\left( \frac{h}{1 - h} \right) = \frac{(1 - \alpha) \beta (1 - \tau^i) \omega(\chi)}{(1 - \theta) \left( \frac{c}{y} \right)}
\]
(II.25)
Aggregate state of the private sector:

\[ k = \left[ \frac{AB^\alpha \bar{\omega}(\chi)}{\left( \frac{k}{y} \right)} \right]^{\gamma(1-\alpha)} h \]  

(II.26)

Government budget constraint:

\[ g_t = \tau_t^k \left[ \alpha \bar{\omega}_t(\chi_t) - \delta \left( \frac{k_t}{y_t} \right) \right] + (1-\alpha)\tau_t^k \tilde{\omega}_t(\chi_t) \]  

(II.27)

where:

\[ \tilde{\omega}_t(\chi_t) \equiv \left[ \frac{1+\frac{1-\alpha\theta}{1-\alpha}}{1+\Delta_t(\chi_t)} \right] \]  

(II.28)
Therefore, in the steady state (II.27) becomes:

\[ g = \tau^k \left[ \alpha \tilde{\omega}(\chi) - \delta \left( \frac{k}{y} \right) \right] + \tau^l (1 - \alpha) \tilde{\omega}(\chi) \]  

(II.29)

**Remark:** Given \( \chi \), only two out of the remaining three government policy variables \( g, \tau^k, \tau^l \) can be chosen independently of each other.

For reasons to be explained in Section IV, we assume that the output share of government consumption is given by:

\[ g_t = \bar{\psi}_t + \psi \chi_t + \frac{1}{2} \psi^2 (\chi_t - \chi_i)^2 ; \bar{\psi}, \psi \geq 0; \]  

(II.30)

where:

\( \bar{\psi} \): output share of conventional government consumption
III UNDERSTANDING THE MODEL

III.1 Effects and Wedges

1. Detailed vs Prototype Economy (Chari, Kehoe, McGrattan (2007))

\[ r_t^* = \alpha \frac{y_t^*}{k_t^*} \]  \hspace{1cm} (III.1)

\[ w_t^* = (1 - \alpha) \frac{y_t^*}{h_t^*} = w_t' \]  \hspace{1cm} (III.2)

\[ \pi_t^* = 0 \]  \hspace{1cm} (III.3)

\[ y_t^* = A_i B_i^o \bar{\omega}_t (\chi_t)^a (h_t^*)^{(1-\alpha)} \]  \hspace{1cm} (III.4)

\[ g_t^* (\chi_t) = \tau_t^{\chi} \left[ \alpha - \bar{\delta} \left( \frac{k_t}{y_t} \right) \right] + (1 - \alpha) (\tau_t^*) \]  \hspace{1cm} (III.5)

where:
\[ \bar{\omega}_i^*(\chi_i) \equiv (1 - \chi_i)^{(1-\theta)/\phi}[1 + \Delta_i^*(\chi_i)]^{(1-\theta)/\phi} \]  

(III.6)

\[ \Delta_i^*(\chi_i) \equiv \left( \frac{B_i'}{B_i'^*} \right)^{(1-\phi)/(1-\phi)} \left( \frac{\chi_i}{1 - \chi_i} \right)^{(1-\theta)/(1-\phi)} \]  

(III.7)

\[ g_i^*(\chi_i) \equiv \bar{\varphi}, \]  

(III.8)
Remark: the laws of motion of the aggregate state of the private sector equilibrium continue to be fully and completely characterized by the intratemporal condition for the household’s labor-leisure choice (I.43), the Euler condition for capital (I.44), the transversality condition (I.45), the resource constraint (II.4) and the government budget constraint (II.19), provided, of course, that the star-superscripted variables, defined above, are used, in the place of their no star-superscripted counterparts. Moreover, private sector equilibrium values of the industry variables, continue to be characterized by (II.5) –(II.10), if we replace \( (\theta, \xi, \Delta, (\chi_t)) \) by \( (1,1,\Delta^*(\chi_t)) \).
Remark: An alternative way to derive the preceding result is to reformulate the model of Sections 1 and 2 so that in the Nash bargaining contract $w^o_t$ is a given reservation wage rate for the union members and reformulate the household problem so that:

$$h_t = \begin{cases} l_t, & \text{if } w^t_t > w^o_t \\ 0, & \text{if } w^t_t \leq w^o_t \end{cases}$$

and then set $w^o_t$ as the opportunity cost of leisure. That is, $w^o_t$ satisfies the intratemporal labor – leisure condition

$$\frac{\theta(1 - \tau^t)}{c_t} = \frac{1 - \theta}{1 - h_t}.$$
Proposition 3: (a) For all \( t \in \mathbb{N}_+ \) and \( \chi_t \in (0,1) \):

\[
\tilde{\omega}_t(\chi_t) : (0,1) \rightarrow (\theta_t, 1), \\
\tilde{\omega}_t(\chi_t) : (0,1) \rightarrow (\xi_t, 1), \\
\tilde{\omega}_t(\chi_t) : (0,1) \rightarrow (0,1) \\
\tilde{\omega}_t(\chi_t) : (0,1) \rightarrow (0,1)
\]

(b) If \( \phi = 0 \), for all \( t \in \mathbb{N}_+ \) and \( \chi_t \in [0,1] \):

\[
\tilde{\omega}_t(\chi_t) = \frac{1 + \theta \left[ \theta_t \xi (1-t) \left( \frac{B_t^i}{B_0^i} \right) \right]^{\gamma(\xi)}}{1 + \xi \left[ \theta_t \xi (1-t) \left( \frac{B_t^i}{B_0^i} \right) \right]^{\gamma(\xi)}} \tag{III.9}
\]

\[
\tilde{\omega}_t(\chi_t) = \frac{1 + \theta \left[ \theta_t \xi (1-t) \left( \frac{B_t^i}{B_0^i} \right) \right]^{\gamma(\xi)}}{1 + \xi \left[ \theta_t \xi (1-t) \left( \frac{B_t^i}{B_0^i} \right) \right]^{\gamma(\xi)}} \tag{III.10}
\]
\[
\frac{\tilde{\omega}_i(\chi_i)}{\omega_i(\chi_i)} = \frac{\left\{ 1 + \left( \theta^\alpha \xi^{(1-\alpha)} \left( \frac{B_i'}{B_i^0} \right) \right)^\gamma_{(1-\alpha)} \right\}^{(1-\alpha)}}{\left[ 1 + \left( \frac{B_i'}{B_i^0} \right)^{1-\alpha} \right]^{(1-\alpha)}}
\]

\[(\text{iii.11})\]

(c) \( \lim_{\chi_i \to 0} \tilde{\omega}_i(\chi_i) = \begin{cases} \theta, & \text{if } \phi > 0 \\ 1, & \text{if } \phi < 0 \end{cases} \) and \( \lim_{\chi_i \to 1} \tilde{\omega}_i(\chi_i) = \begin{cases} \theta, & \text{if } \phi > 0 \\ 1, & \text{if } \phi < 0 \end{cases} \)

\( \lim_{\chi_i \to 0} \tilde{\omega}_i(\chi_i) = \begin{cases} \xi, & \text{if } \phi > 0 \\ 1, & \text{if } \phi < 0 \end{cases} \) and \( \lim_{\chi_i \to 1} \tilde{\omega}_i(\chi_i) = \begin{cases} \xi, & \text{if } \phi > 0 \\ 1, & \text{if } \phi < 0 \end{cases} \)

\( \lim_{\chi_i \to 0} \frac{\tilde{\omega}_i(\chi_i)}{\omega_i(\chi_i)} = \begin{cases} 1, & \text{if } \phi > 0 \\ 0, & \text{if } \phi < 0 \end{cases} \) and \( \lim_{\chi_i \to 1} \frac{\tilde{\omega}_i(\chi_i)}{\omega_i(\chi_i)} = \begin{cases} 0, & \text{if } \phi > 0 \\ 1, & \text{if } \phi < 0 \end{cases} \)

(d) \( \tilde{\omega}_i'(\chi_i), \omega_i'(\chi_i) = \begin{cases} < 0, & \text{if } \phi > 0 \\ > 0, & \text{if } \phi < 0 \end{cases} \)
Remark: It follows that the capital wedge, $\omega(\chi)$, is less than one, in general, and decreasing (increasing) with \( \chi \) if insiders’ and outsiders’ sector inputs, in final good production, are substitutes (complements). The reason, this is happening is that the demand for capital is lower in the detail economy than in the prototype economy for two reasons (i) the monopolistic producer restricts output needing less from both inputs, (ii) the firm-union bargaining results in a higher wage rate and further reduces labor input in such a way that the output effect dominates the substitution effect. Further, if insiders’ and outsiders’ sector inputs in the production of the final good are closed substitutes (no closed substitutes) the demand for capital in insiders’ industries will be lower (higher), as the marginal product of capital is lower (higher).

The main implication of this distortion is to lower the marginal product of capital in the economy and lower it relatively more (less) if insiders’ and outsiders’ sector inputs in the production of the final good are closed substitutes (no closed substitutes). This has a static and a dynamic consequence. The static consequence is immediate from the steady state relationship (II.23) and implies a lower steady state capital-output ratio for the economy. The dynamic consequence follows from the household Euler condition for capital (I.44) and the equilibrium equation for the rental cost of capital rental cost equation (II.12):

$$u_{c_t}^h = -\frac{\beta}{(1+\eta)}E_t\left\{u_{c_{t+1}}^h [1+(1-\tau^h)][(\alpha\omega_{t+1}\chi_{t+1})^{\gamma_{t+1}} - \delta]\right\} \quad \text{(III.14)}$$

That is, in the detailed economy investment, in any given period \( t \), must equate the marginal value of sacrificing current consumption to a lower discounted expected marginal value of next period consumption due to the after tax gross return of this investment, than in the prototype economy.
Qualitatively, the labor wedge behaves much the same and for similar reasons like the capital wedge. The main implication of the labor wedge is the distortion of the equality of the MRS between consumption and leisure and the corresponding MRT in the economy. This is immediate from the household labor-leisure intratemporal condition (I.43) and the wage equations (II.13) and (III.2):

\[
\frac{u_t^b}{u_t^c} = \omega_t h_t \left(1 - \alpha_t \right) \frac{V_t}{h_t}
\]

(III.15)

Thus, the detailed economy has a lower MRT than the prototype economy. This again has a static and a dynamic consequence. The static consequence is immediate from the steady state version of the preceding equation (i.e., (69)) and implies lower employment in the steady state of the detailed economy than in the prototype economy. The dynamic consequences are apparent from (III.9) and (III.10).

*Efficiency Wedge*: \(\frac{\partial \varphi_t(\chi_t)}{\partial \chi_t} \) is less than one, for the reasons explained above. Also, it is smaller (larger) when \(\phi \) is relatively large (small). Inverted “U” shape.

**III.2 Sensitivity Analysis of PSE Steady State**
The main idea for the calibration is to think of the US as the prototype (star) economy and the South European economies as detailed economies for different (insider-outsider society) parameter values.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>capital input elasticity in industry production</td>
<td>(1/3) (Calibrated from US data (\alpha = \frac{r^<em>k^</em>}{y^*} = 1/3))</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>constant discount factor of households</td>
<td>0.97</td>
<td>SVL</td>
</tr>
<tr>
<td>(1/\gamma)</td>
<td>household intertemporal elasticity of substitution</td>
<td>0.5</td>
<td>SVL</td>
</tr>
<tr>
<td>(\delta)</td>
<td>capital depreciation rate</td>
<td>0.07</td>
<td>SVL</td>
</tr>
<tr>
<td>(\eta)</td>
<td>growth rate of labor of augmenting technology</td>
<td>0.02</td>
<td>SVL</td>
</tr>
<tr>
<td>(\theta)</td>
<td>intensity of consumption in household preferences</td>
<td>(1/3)</td>
<td>SVL</td>
</tr>
<tr>
<td>(\frac{1}{1-\theta})</td>
<td>aggregation elasticity of substitution across industries in the same sector</td>
<td>10</td>
<td>CO</td>
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<tr>
<td>(\lambda)</td>
<td>intensity of wage premium in union preferences</td>
<td>Calibrated from (17) and (21) such that: (\frac{w^l}{w^o} = 1.1)</td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Value/Range</td>
<td>Source</td>
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<tr>
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<tr>
<td>$\mu$</td>
<td>relative bargaining power of insiders’ unions</td>
<td>1</td>
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</tr>
<tr>
<td>$\tau^k$</td>
<td>capital income tax rate</td>
<td>0.2</td>
<td>Data</td>
</tr>
<tr>
<td>$\frac{1}{1-\phi}$</td>
<td>elasticity of substitution across sectors in final good production</td>
<td>-0.9, 0.1, 0.9</td>
<td></td>
</tr>
<tr>
<td>$\chi$</td>
<td>fraction of insiders industries in the economy</td>
<td>[0.1,0.9]</td>
<td></td>
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<tr>
<td>$\bar{\psi}$</td>
<td>GDP share of government consumption in the US</td>
<td>0.225</td>
<td>Data</td>
</tr>
<tr>
<td>$\psi$</td>
<td>such that the difference between the GDP share of government consumption in South Europe and the US is $\psi \chi$</td>
<td>0.05</td>
<td>Data</td>
</tr>
</tbody>
</table>
PSE Steady State Sensitivity with respect to chi / phi=-0.9

![Graph 1: Y vs. Chi/Phi](image1)

![Graph 2: Y' vs. Chi/Phi](image2)

![Graph 3: C vs. Chi/Phi](image3)

![Graph 4: C' vs. Chi/Phi](image4)
Sensitivity with respect to chi / phi=-0.9 (continued)
Sensitivity with respect to chi / phi=-0.9 (continued)
Sensitivity with respect to chi / phi=-0.9 (continued)
Sensitivity with respect to \( \chi / \phi = -0.9 \) (continued)
Sensitivity with respect to chi / phi=0.1
Sensitivity with respect to $\chi / \phi = 0.1$ (continued)
Sensitivity with respect to chi / phi=0.1 (continued)
\( \frac{\pi}{\rho_0} \)

\( r \)

\( \rho \)

\( \frac{(\pi/\rho_0)^*}{\rho_0} \)

\( r^* \)

\( \rho \)

\( \rho^* \)

\( \omega \)

\( \omega^* \)
Sensitivity with respect to \( \chi / \phi = 0.1 \) (continued)
Sensitivity with respect to chi / phi=0.1 (continued)
Sensitivity with respect to $\chi / \phi=0.9$
Sensitivity with respect to chi / phi=0.9 (continued)
Sensitivity with respect to chi / phi=0.9 (continued)
\[
\frac{\pi}{\rho_0}
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\[
0.14313809761768
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\[
0.14313809761766
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Sensitivity with respect to chi / phi=0.9 (continued)
Sensitivity with respect to chi / phi=0.9 (continued)
III.3 PSE IRFs

Impulse Response Functions / Prototype Economy/ phi=-0.9
Shock to A
Impulse Response Functions / Prototype Economy/ phi=-0.9
Shock to chi
Impulse Response Functions / Prototype Economy/ phi=0.1
Shock to A
Impulse Response Functions / Prototype Economy/ $\phi=0.1$
Shock to $\chi$
Impulse Response Functions / Prototype Economy/ $\phi=0.9$
Shock to A
Impulse Response Functions / Prototype Economy/ phi=0.9
Shock to chi
Impulse Response Functions / Detailed Economy/ phi=-0.9
Shock to A
Impulse Response Functions / Detailed Economy/ phi=-0.9
Shock to chi
Impulse Response Functions / Detailed Economy/ phi=-0.9
Shock to chi (continued)
Impulse Response Functions / Detailed Economy/ \( \phi=0.1 \)

Shock to A

<table>
<thead>
<tr>
<th>Variable</th>
<th>Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td><img src="image1" alt="Graph of y" /></td>
</tr>
<tr>
<td>( c )</td>
<td><img src="image2" alt="Graph of c" /></td>
</tr>
<tr>
<td>( h )</td>
<td><img src="image3" alt="Graph of h" /></td>
</tr>
<tr>
<td>( k )</td>
<td><img src="image4" alt="Graph of k" /></td>
</tr>
<tr>
<td>( w_0 )</td>
<td><img src="image5" alt="Graph of w_0" /></td>
</tr>
<tr>
<td>( r )</td>
<td><img src="image6" alt="Graph of r" /></td>
</tr>
<tr>
<td>( u_h )</td>
<td><img src="image7" alt="Graph of u_h" /></td>
</tr>
<tr>
<td>( w_0 )</td>
<td><img src="image8" alt="Graph of w_0" /></td>
</tr>
<tr>
<td>( A )</td>
<td><img src="image9" alt="Graph of A" /></td>
</tr>
</tbody>
</table>
Impulse Response Functions / Detailed Economy/ phi=0.1
Shock to chi
Impulse Response Functions / Detailed Economy/ phi=0.1
Shock to chi (continued)
Impulse Response Functions / Detailed Economy/ \( \phi = 0.9 \)

Shock to A

\[ \text{y, c, h, wo, r, tauL} \]

96
Impulse Response Functions / Detailed Economy/ \( \phi=0.9 \)

Shock to \( \chi \)
Impulse Response Functions / Detailed Economy/ phi=0.9
Shock to chi (continued)
III. Politicoeconomic Equilibrium

1. Government Policy Variables

We assume that the capital income tax rate, $\tau^c$, and $\tau^L$ follow are exogenous. Thus, the only endogenous government policy variables are the labor tax rate, $\tau^L$, and the fraction of insiders’ industries in the economy, $\chi_{i+1}$.

2. Definition (Persson and Tabellini (2002, Ch.4))

A politicoeconomic equilibrium is a sequence of the form $\{c_t(z'), h_t(z'), k_{t+1}^*(z'), \tau_t^L(z'), \chi_{t+1}^*(z')\}_t^\infty$, such that:

$$\{c_t(z'), h_t(z'), k_{t+1}^*(z'), \tau_t^L(z'), \chi_{t+1}^*(z')\}_t^\infty = \arg \min_{\{c_t(z'), h_t(z'), k_{t+1}^*(z'), \tau_t^L(z'), \chi_{t+1}^*(z')\}_t^\infty} U_0^\rho$$

where:

$$U_0^\rho = \rho \frac{\overline{U}_0^l - U_0^\rho}{U_0^l} + (1 - \rho) \frac{\overline{U}_0^h - U_0^h}{U_0^h}; \rho \in [0,1]$$
\[ U_0^i = E_0 \sum_{t=0}^{\infty} \beta^t \chi_i^{(1-\lambda)} u_i \]  \hspace{1cm} (IV.3)

\[ \overline{U}_0^i = \max_{\{h_t (z'), k_{t+1} (z'), \tau_{t+1} (z'), \chi_{t+1} (z')\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \chi_i^{(1-\lambda)} u_i \]  \hspace{1cm} (IV.4)

\[ \overline{U}_0^h = \max_{\{h_t (z'), k_{t+1} (z'), \tau_{t+1} (z'), \chi_{t+1} (z')\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u_i^h \]  \hspace{1cm} (IV.5)

and; where the sequences \{h_t (z'), k_{t+1} (z'), \tau_{t+1} (z'), \chi_{t+1} (z')\}_{t=0}^{\infty} in all the above optimization problems are restricted to obey the equilibrium laws of motion of the aggregate state of the private sector, defined in Proposition 2 and the initial condition:

\[(k_0, \chi_0) \in (0, \infty) \times [0,1] \text{ given.}\]
Remark: (a) If $\rho = 0$, the government objective function is that of the representative household (i.e., median voter). In this case, government behaves like a Ramsey planner. If $\rho = 1$, the government objective function is that of a union comprising of all insiders in the economy. In general, $\rho$ is a parameter that captures the influence of insiders in government decision making. (b) Evidently, for $U_0^\rho$ to be well defined, we must have $|U_0^\rho|, |\bar{U}_0^\rho| < +\infty$. Initially, we simply assume this, but subsequently we show that in the steady state these conditions are satisfied.

Proposition 4: Suppose that \( \overline{U}_0, \overline{U}_0^b \in \mathbb{R} \), \( \overline{U}_0^b < +\infty \). Then, there exists a real number \( m \in \mathbb{R} \), such that if \( \{c_i^*(z^i), h_i^*(z^i), k_{t+1}^*(z^i), \chi_{t+1}^*(z^i)\}_{z^i} \) is a politicoeconomic equilibrium, the following are true:

(A) \( \{c_i^*(z^i), h_i^*(z^i), k_{t+1}^*(z^i), \chi_{t+1}^*(z^i)\}_{z^i} \) is a solution to:

\[
\max_{\{c_i^*(z^i), h_i^*(z^i), k_{t+1}^*(z^i), \chi_{t+1}^*(z^i)\}_{z^i}} \left\{ E_0 \sum_{t=0}^{\infty} \beta^t M \left[ c_t(z^i), h_t(z^i), k_{t+1}(z^i), \chi_{t+1}(z^i), z_i \right] \right\} + N \left[ c_0(z_0), h_0(z_0), k_0, \chi_0, z_0 \right]
\]

subject to the resource constraint:

\[
[1 - \bar{\psi}_t - \psi_t \chi_t - \frac{1}{2} \dot{\psi}(\chi_{t+1} - \chi_t)^2] \bar{\omega}_t(\chi_t) k_t^{\alpha} h_t^{(1-\alpha)} = c_t + [(1 + \eta) k_{t+1} - (1 - \delta) k_t] \]

and the initial condition:

\( (k_0, \chi_0) \in (0, \infty) \times (0,1) \) given

(B) The labor tax policy is set according to:

\[
\tau_t^L = \frac{\bar{\psi}_t + \psi_t \chi_t + \frac{1}{2} \dot{\psi}(\chi_{t+1} - \chi_t)^2 - \tau_t^k}{(1 - \alpha) \bar{\omega}_t(\chi_t)} \left[ \alpha \bar{\omega}_t(\chi_t) - \delta \left( \frac{k_t}{y_t} \right) \right]
\]
where (condensing notation):

\[ M(c_t, h_t, k_{t+1}, \lambda_{t+1}, z_t) = \hat{u}_t \{1 - \hat{m}[1 - \hat{\omega}_t(\lambda_t) \frac{(1 - \delta)h_t}{\delta(1 - h_t)}]\} + \hat{u}_t \]  

(IV.10)

\[ N(c_o, h_0, k_0, \lambda_0, z_0) = m[1 + (1 - \tau_o^k)(r_0 - \delta)] \frac{\mu_0^k}{c_0^k} k_0 \]  

(IV.11)

\[ \hat{u}_t \equiv \left[ \frac{\rho U_0^h}{(1 - \rho)U_0^i} \right] \chi_t^{(1 - \alpha)} u_i^{(1 - \alpha)} = \hat{\omega}_t(\lambda_t) \hat{\omega}_i(\lambda_t) \lambda_i k_i \alpha_i h_i^{(1 - \alpha_i)} \]  

(IV.12)

\[ \hat{\omega}_t(\lambda_t) \equiv \left[ \frac{\rho U_0^h}{(1 - \rho)U_0^i} \right] \left[ \frac{(1 - \alpha)(\nu - 1)^{i + 1} \hat{\omega}_i(\lambda_t) \hat{\omega}_i(\lambda_t)}{[1 + \hat{\omega}_i(\lambda_t)]^2 \hat{\omega}_i(\lambda_t) \hat{\omega}_i(\lambda_t)} \right] \]  

(IV.13)

1st o. c.

The set of first order conditions for the problem of the government consists of the implementability constraint (90) and the following:
For $t = 0$:

\[ M_{k_0} + N_{k_0} = (1 - \alpha)(1 - g_0)\bar{\alpha}_0(\chi_0) \left( \frac{k_0}{h_0} \right)^\alpha (M_{c_0} + N_{c_0}) \]  

(IV.17)

\[
(1 + \eta)(M_{c_0} + N_{c_0}) = M_{k_i} + \beta E_0 \left\{ M_{c_i} \left[ (1 - \delta) + \alpha(1 - g_i)\bar{\alpha}_i(\chi_i) \left( \frac{k_i}{h_i} \right)^{a-1} \right] \right\}
\]  

(IV.18)

\[
(M_{c_0} + N_{c_0})\bar{\psi}(\chi_i - \chi_0)\bar{\alpha}_0(\chi_0)k_0^{1-\alpha} = M_{\chi_i} + \beta E_0 \left\{ M_{c_i} \left[ -\bar{\psi} + \bar{\psi}(\chi_2 - \chi_1) + (1 - g_i)\frac{\bar{\alpha}_i'(\chi_i)}{\bar{\alpha}_i(\chi_i)} \right] \bar{\alpha}_i(\chi_i)k_i^{1-\alpha} \right\}
\]  

(IV.19)

For $t > 0$:

\[ M_{k_i} = (1 - \alpha)(1 - g_i)\bar{\alpha}_i(\chi_i) \left( \frac{k_i}{h_i} \right)^\alpha M_{c_i} \]  

(IV.20)

\[
(1 + \eta)M_{c_i} - M_{k_{i+1}} = \beta E_i \left\{ M_{c_{i+1}} \left[ (1 - \delta) + \alpha(1 - g_{i+1})\bar{\alpha}_{i+1}(\chi_{i+1}) \left( \frac{k_{i+1}}{h_{i+1}} \right)^{a-1} \right] \right\}
\]  

(IV.21)

\[
M_{c_i}\bar{\psi}(\chi_{i+1} - \chi_i)\bar{\alpha}_i(\chi_i)k_i^{1-\alpha} - M_{\chi_{i+1}} = \beta E_i \left\{ M_{c_{i+1}} \left[ -\bar{\psi} + \bar{\psi}(\chi_{i+2} - \chi_{i+1}) + (1 - g_{i+1})\frac{\bar{\alpha}_{i+1}'(\chi_{i+1})}{\bar{\alpha}_{i+1}(\chi_{i+1})} \right] \bar{\alpha}_{i+1}(\chi_{i+1})k_{i+1}^{1-\alpha} \right\}
\]  

(IV.22)
IV.2 PE Asymptotic Steady State

We consider the asymptotic steady state \((c, h, k, \chi, m)\) implied by the first-order conditions for \(t > 0\) and the implementability constraint (IV.16) for \((k_0, \chi_0) = (k, \chi)\). In order to condense notation, we also use steady state output, \(y\), and steady state output share of government expenditures, \(g\), where: 
\[
y = \bar{\alpha}(\chi)k^\alpha h^{(1 - \alpha)} \quad \text{and} \quad g = \bar{\psi} + \hat{\psi}\chi. \]

Then, it follows that the asymptotic steady state is characterized by:
\[
\left(\frac{M_h}{M_c}\right) = (1 - \alpha)(1 - g)\left(\frac{y}{h}\right) \quad \text{(IV.23)}
\]
\[
[\beta^{-1}(1 + \eta) - (1 - \delta)] - \beta^{-1}\left(\frac{M_h}{M_c}\right) = \alpha(1 - g)\left(\frac{y}{k}\right) \quad \text{(IV.24)}
\]
\[
\beta^{-1}\left(\frac{M_h}{M_c}\right) = \tilde{\psi} - (1 - g)\frac{\partial(\chi)}{\partial(\chi)} \quad \text{(IV.25)}
\]
\[
(1 - \beta)^{-1}[1 - \bar{\omega}(\chi)]\left(\frac{1 - \delta}{\delta(1 - h)}\right)\left(\frac{c}{y}\right) - [1 - (1 - \tau^k)\delta]\left(\frac{k}{y}\right) = a(1 - \tau^k)\bar{\omega}(\chi) \quad \text{(IV.26)}
\]
\[
\left( \frac{c}{y} \right) + (\eta + \delta) \left( \frac{k}{y} \right) = (1 + g)
\]  

where:

\[
\left( \frac{M_h}{M_c} \right) = \frac{(1-\vartheta)c}{\vartheta(1-h)} \left\{ 1 + \frac{m\ddot{\omega}(\chi) - \frac{\ddot{u}^\prime}{u^\prime}}{1 - m[1 - \ddot{\omega}(\chi)\frac{(1-\vartheta)h}{\vartheta(1-h)}]} \right\}
\]  

\[
\left( \frac{M_k}{M_c} \right) = \frac{c}{\vartheta} \frac{\left( \frac{\ddot{u}^\prime}{u^\prime} \right) \alpha \lambda}{k}
\]  

\[
\left( \frac{M_f}{M_c} \right) = \frac{m\ddot{\omega}(\chi)\frac{(1-\vartheta)h}{\vartheta(1-h)} + \left( \frac{\ddot{u}^\prime}{u^\prime} \right) \left( \ddot{\omega}(\chi) - \lambda \ddot{\omega}(\chi) \right)}{1 - m[1 - \ddot{\omega}(\chi)\frac{(1-\vartheta)h}{\vartheta(1-h)}]}
\]  

(IV.27) (IV.28) (IV.29) (IV.30)
Note that (IV.30) can be solved for $m$, to get:

$$m = \frac{(1-\alpha)(1-g)\left(\frac{y}{\bar{h}}\right)}{(1-\vartheta)c} - 1 - \frac{\bar{u}'}{\bar{k}} \frac{\vartheta^2(1-\alpha \lambda)(1-h)}{(1-\vartheta)h}$$

Eliminating the multiplier, in (IV.24)-(IV.27) are four equations that define the asymptotic steady state in terms of $c, h, k, \chi$.

Remark: To compute $\bar{U}_0^h$ and $\bar{U}_0^i$ simply note that $U_0^\rho = \bar{U}_0^h$, for $\frac{\rho \bar{U}_0^h}{(1-\rho)\bar{U}_0^i} = 0$ and $U_0^\rho = \bar{U}_0^i$ for $\frac{\rho \bar{U}_0^h}{(1-\rho)\bar{U}_0^i} = 1$ and $M(c, h, k, z, \chi, \vartheta) = -m[1 - \bar{u}^\prime(\chi')](1-\vartheta)\bar{k} + \bar{u}^\prime$. The preceding results suggest the method described in the next section, to compute the politicoeconomic equilibrium.
IV. 3 Computing the Politicoeconomic Equilibrium

1. Algorithm for Computing the Politicoeconomic Equilibrium:

Step 1: Compute the asymptotic steady state of the politicoeconomic equilibrium in the Ramsey planner case, using equations (IV.31) and (IV.24)-(IV.27), to compute $\bar{U}_0^h$, setting $\frac{\rho \bar{U}_0^h}{(1 - \rho) \bar{U}_0^e} = 0$. 
Step 2: Compute the asymptotic steady state of the politicoeconomic equilibrium in the insiders’ government case, using equations (IV.31) and (IV.24)-(IV.27), to compute $\bar{U}_0^i$, setting $\frac{\rho \bar{U}_0^h}{(1 - \rho) \bar{U}_0^i} = 1$ and

$$M(c_t, h_t, \chi_{t+1}, z_t) = -m[1 - \bar{\omega}_t(\chi_t) \frac{(1 - \vartheta) h_t}{\vartheta(1 - h_t)}] \mu_t^h + \hat{u}_t^i$$

Step 3: Compute the asymptotic steady state of the politicoeconomic equilibrium in the general case, for any given $\rho \in (0, 1)$, using equations (IV.31) and (IV.24)-(IV.27). Note, that this implies that the government objective function minimizes a weighted average of deviations of the Ramsey planner and the insiders’ government from their respective steady states, which is different from these deviations from any given initial condition $(k_0, \chi_0) \in (0, \infty) \times [0, 1]$.

Step 4: Assume that in period 0 the politicoeconomic equilibrium is in its asymptotic steady state computed in Step 3 and obtain the solution as an approximation to this asymptotic steady state.

Step 1: Compute the asymptotic steady state of the politicoeconomic equilibrium in the Ramsey planner case, using equations (IV.31) and (IV.24)-(IV.27), to compute $\bar{U}_0^h$, setting $\frac{\rho \bar{U}_0^h}{(1 - \rho) \bar{U}_0^i} = 0$. 

111
Step 2: Compute the asymptotic steady state of the politicoeconomic equilibrium in the insiders’ government case, using equations (IV.31) and (IV.24)-(IV.27), to compute $\bar{U}_0^i$, setting $\rho \bar{\bar{U}}_0^h (1 - \rho) \bar{U}_0^i = 1$ and

$$M(c_i, h_i, k_{i+1}, x_{i+1}, z_i) = -m[1 - \hat{\omega}_i(\chi_i, (1 - \tau)h_i, \frac{\theta(1 - h_i)}{(1 - h_i)}) \mu_e^h + \hat{\mu}_e^i]$$