The Historical Evolution of the Wealth Distribution: A Quantitative-theoretic Investigation

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Evolution of top wealth inequality in the U.S.

Overview: objective

- calibrate a quantitative macro model that accounts for the full U.S. wealth distribution, including the Pareto tail
- study the transition path: starting in 1960s, feeding in observed changes in earnings inequality and tax rates
- can the standard macro-inequality framework explain movements in the wealth distribution?
Overview: findings

- baseline model is successful in explaining the long-run evolution of the wealth distribution
  - cumulative increase in inequality explained well for bulk of distribution
- active channels:
  - decreasing tax progressivity has a dramatic effect on the wealth distribution
  - increase in idiosyncratic labor income risk has a dampening effect on wealth inequality via precautionary savings channel
  - changes in $r - g$ not important, partly working in the opposite direction
- misses short-run dynamics: short-run movements in inflation and asset return help to account for the U-shape
- cautious prediction for 21st century: long-term effects of decreasing tax progressivity on wealth inequality
Trends in wealth inequality: recent literature

Quantitative model

- Aiyagari 1994 framework:
  - log labor income as sum of persistent and transitory component; adjusted at the top to match the observed Pareto tail in labor income
  - transitory component incorporates zero earnings state
  - stochastic discount factor follows AR(1) process (Krusell-Smith 1998 extended); for genetic evidence, see Cronqvist and Siegel 2015 on “the origins of savings behavior”
  - stochastic i.i.d. return on capital
  - progressive taxation: use data on federal effective tax rates for 11 income brackets (Piketty & Saez 2007)
  - parsimonious modeling of social safety net: 60% of tax revenues rebated as lump-sum transfers
- time-varying tax system and labor income process
The consumer’s problem

\[ V_t(x_t, p_t, \beta_t) = \max_{a_{t+1} \geq a} \{u(x_t - a_{t+1}) + \beta_t \mathbb{E}[V_{t+1}(x_{t+1}, p_{t+1}, \beta_{t+1})|p_t, \beta_t]\} \quad (1) \]

subject to:
\[ x_{t+1} = a_{t+1} + y_{t+1} - \tau_{t+1}(y_{t+1}) + T_{t+1} \quad (2) \]
\[ y_{t+1} = r_{t+1} \eta_{t+1} a_{t+1} + w_{t+1} l_{t+1}(p_{t+1}, \nu_{t+1}) \quad (3) \]

\( x_t \) cash on hand
\( p_t \) persistent component of earnings process
\( l_{t+1}(\cdot, \cdot) \) efficiency units of labor, moves over time
\( \nu_{t+1} \) transitory earnings shock
\( \eta_{t+1} \) return to capital shock
\( \tau_t(y_t) \) tax function based on gross income, moves over time
\( T_t \) lump-sum transfer
Main qualitative mechanism (at the top)

- stochastic-$\beta$ alone generates a Pareto tail in the wealth distribution
  - add stochastic return to capital and Pareto tail in labor income to improve quantitative properties of the model
  - Pareto tail in labor income alone would be inherited by wealth distribution, but tail coefficient would be too high (top inequality too low)
- follows from random growth theory (Kesten 1973, see also Gabaix 2009)
  - mechanism has been employed by Benhabib, Bisin and Zhu 2015, Nirei & Aoki 2015, Piketty & Zucman 2015
Stochastic-β yields stochastic, linear savings decisions

- high beta, high earnings
- high beta, low earnings
- low beta, high earnings
- low beta, low earnings

![Graph showing marginal propensity to save against log(k)]
Gives rise to a Pareto tail in the wealth distribution.
Calibration strategy

- earnings process, tax rates, social safety net calibrated to observables
- randomness in discount factor and return to capital calibrated to replicate the wealth distribution in the initial steady state (1967)
- focus on tail coefficient alone misleading: even if say the richest 10% can be described exactly by a Pareto distribution, the shape parameter only tells us how wealth is distributed within these 10%, not how much wealth the top 10% control as a fraction of total wealth
Calibration

**Stochastic-β:**
- follows AR(1) process with: $\mu = 0.92$, $\rho = 0.992$, $\sigma = 0.0019$
- in cross-section, standard deviation = 0.0148; over 50 years, mean reversion is $1/3$

**Stochastic return to capital:**
- pre-tax return $(1 + r_t \eta_t)$ with $\eta_t \sim iidN(1, 0.725)$
- in steady state, standard deviation of 0.048 or 90% have return $(1 + r^*\eta_t) \in [0.9874, 1.1437]$
- Fagereng, Guiso, Malacrino & Pistaferri 2016 find a standard deviation of 0.04 in Norwegian data

**EIS and aggregate technology:**
- CRRA = 1.5
- Cobb-Douglas with capital’s share = 0.36; $\delta = 0.048$
### Matching the wealth distribution

#### U.S. wealth distribution in 1967:

<table>
<thead>
<tr>
<th></th>
<th>Top 10% Share</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong>*</td>
<td>70.8%</td>
<td>27.8%</td>
<td>9.4%</td>
<td>3.1%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>70.6%</td>
<td>28.1%</td>
<td>9.5%</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>fraction w negative wealth</th>
<th>Bottom 50% share</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong>*</td>
<td>8.0%</td>
<td>4.0%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td>7.0%</td>
<td>3.1%</td>
</tr>
</tbody>
</table>


- probability of zero earnings state (0.075) and borrowing constraint (approx. equal to the annual transfer) chosen to match left tail
- model matches wealth distribution well on its entire domain
Observed change 1: decrease in tax progressivity

- federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes
Observed change 2: increase in labor income risk

Observed change 3: increase in top labor income shares

- adjust standard AR(1) in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (Piketty & Saez 2003 [updated series in 2011])
Main result: Cumulative Change in Top Wealth Shares

<table>
<thead>
<tr>
<th></th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>70.8</td>
<td>27.8</td>
<td>9.4</td>
<td>3.1</td>
</tr>
<tr>
<td>2012</td>
<td>77.2</td>
<td>41.8</td>
<td>22.0</td>
<td>11.2</td>
</tr>
<tr>
<td>Rel. Δ</td>
<td>9.0%</td>
<td>50.4%</td>
<td>134.0%</td>
<td>261.3%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>70.6</td>
<td>28.1</td>
<td>9.5</td>
<td>2.9</td>
</tr>
<tr>
<td>2012</td>
<td>79.0</td>
<td>37.3</td>
<td>13.4</td>
<td>4.2</td>
</tr>
<tr>
<td>Rel. Δ</td>
<td>11.9%</td>
<td>32.5%</td>
<td>41.0%</td>
<td>45.8%</td>
</tr>
<tr>
<td><strong>Fraction explained</strong></td>
<td>132.2%</td>
<td>64.6%</td>
<td>30.6%</td>
<td>17.5%</td>
</tr>
</tbody>
</table>

Wealth shares in %.
... when compared to SCF data

<table>
<thead>
<tr>
<th></th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>67.1</td>
<td>30.1</td>
<td>10.8</td>
</tr>
<tr>
<td>2013</td>
<td>75.3</td>
<td>35.8</td>
<td>13.5</td>
</tr>
<tr>
<td>Rel. ∆</td>
<td>12.2%</td>
<td>19.1%</td>
<td>25.4%</td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>73.4</td>
<td>30.4</td>
<td>10.2</td>
</tr>
<tr>
<td>2013</td>
<td>79.2</td>
<td>37.5</td>
<td>13.5</td>
</tr>
<tr>
<td>Rel. ∆</td>
<td>8.0%</td>
<td>23.6%</td>
<td>32.7%</td>
</tr>
<tr>
<td><strong>Fraction explained</strong></td>
<td>65.3%</td>
<td>123.5%</td>
<td>128.6%</td>
</tr>
</tbody>
</table>

Wealth shares in %

Data: Survey of Consumer Finance (SCF), as reported by Saez & Zucman (2016).
### Other Results

<table>
<thead>
<tr>
<th></th>
<th>Bottom 50%</th>
<th>personal wealth</th>
<th>national w.</th>
<th>$K_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>4.0%</td>
<td>3.6</td>
<td>4.1</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>1.1%</td>
<td>4.1</td>
<td>4.6</td>
<td></td>
</tr>
<tr>
<td>Rel. $\Delta$</td>
<td>-73%</td>
<td>14%</td>
<td>14%</td>
<td></td>
</tr>
<tr>
<td><strong>Model</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1967</td>
<td>3.1%</td>
<td></td>
<td>3.7</td>
<td></td>
</tr>
<tr>
<td>2010</td>
<td>1.4%</td>
<td></td>
<td>4.2</td>
<td></td>
</tr>
<tr>
<td>Rel. $\Delta$</td>
<td>-55%</td>
<td></td>
<td>13%</td>
<td></td>
</tr>
<tr>
<td><strong>Fraction explained</strong></td>
<td>76%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Bottom 50% Data: Survey of Consumer Finance (SCF), as reported by Kennickell (2011).

Personal/ National wealth data: Piketty & Zucman (2014)
Summary of transitional dynamics

- model captures the salient features of the evolution of the U.S. wealth distribution
- assumptions that we found are not critical:
  - perfect foresight (myopic transition)
  - robust to CES production function with elasticity $> 1$
- shortcomings:
  - explosion of wealth concentration at the very top (0.1% and above) as measured by Saez & Zucman (2016) not explained well
  - miss on short-run dynamics (heterogeneous portfolios and valuation effects? time-varying inflation?): more on this later
Main channels

what fraction of the increase in the top wealth shares do the three channels account for?

<table>
<thead>
<tr>
<th></th>
<th>Earnings Risk</th>
<th>Top Earnings</th>
<th>Taxes</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10%</td>
<td>−0.78</td>
<td>0.22</td>
<td>1.89</td>
<td>1.32</td>
</tr>
<tr>
<td>Top 1%</td>
<td>−0.19</td>
<td>0.05</td>
<td>0.82</td>
<td>0.65</td>
</tr>
</tbody>
</table>

larger earnings risk induces higher precautionary savings
  especially among the less wealthy, reducing tendency of heterogeneous discount factors to drive apart the wealth distribution (Becker 1980)
  interest rate falls, thereby increasing the Pareto tail coefficient (i.e., decreasing top wealth inequality)

decreasing progressivity of taxes has a large effect
  intuition: under complete markets, strict tax progressivity implies full equality, without progressivity indeterminacy
What about $r - g$?

- Increase in $r - g$ decreases wealth inequality in the medium run (a few decades).
- Pareto tail coefficient decreases (i.e., top wealth inequality increases), but very slowly.
  - Random growth models generally feature slow transitions: it takes long to fill a thick long tail (see Gabaix, Lasry, Lions, and Moll 2016).
- More important in short run: low-asset agents' savings decisions more elastic w.r.t. the interest rate.
$r - g$ experiment

- **pre-tax interest rate**
  - Year 0: 10%
  - Year 20: 6%
  - Year 40: 7%
  - Year 60: 8%
  - Year 80: 9%
  - Year 100: 10%

- **top 1% wealth share**
  - Year 0: 25%
  - Year 20: 26%
  - Year 40: 27%
  - Year 60: 28%
  - Year 80: 29%

- **Gini Coefficient for Wealth**
  - Year 0: 0.78
  - Year 20: 0.79
  - Year 40: 0.8
  - Year 60: 0.81
  - Year 80: 0.82
  - Year 100: 0.83

- **Gini Coefficient for Income**
  - Pre-tax income:
    - Year 0: 0.3
    - Year 20: 0.35
    - Year 40: 0.4
    - Year 60: 0.45
    - Year 80: 0.5
  - Post-tax income:
    - Year 0: 0.45
    - Year 20: 0.4
    - Year 40: 0.35
    - Year 60: 0.3
    - Year 80: 0.35
    - Year 100: 0.3
Accounting for short-run dynamics

- lots of evidence that asset portfolios vary systematically with wealth
  - ... and risky asset prices move substantially (valuation effects)
- inflation affects real return to capital because nominal capital income is taxed
  - ... and inflation moves substantially over the sample period
  - the real after-tax return $\tilde{r}$ is given by
    \[ \tilde{r} \approx r(1 - \tau) - \tau \pi \]
- assess quantitative importance of both of these channels simply by “adding” them to the baseline transition path (avoids taking a stand on how to model portfolio choice, or on why asset prices move)
Heterogeneous portfolios

Risky Asset Portfolio Weights

Valuation effects

Annual Real Returns (1967 = average 1960-67)

- Real Interest Rate
- Real S&P Return
- Equity Premium
Heterogeneous portfolios and valuation effects: top shares
Time-varying inflation

![Graph showing CPI Inflation](image)
Time-varying inflation: top shares

**Top 10% Wealth Share**
- **Benchmark Model**
- **Benchmark + Inflation**
- **Data (SZ)**
- **Data (SCF)**

**Top 1% Wealth Share**

**Top 0.1% Wealth Share**

**Top 0.01% Wealth Share**
Capital in the 21st century?

<table>
<thead>
<tr>
<th>Year</th>
<th>Top 10%</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
<th>Bottom 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1967</td>
<td>70.6</td>
<td>28.1</td>
<td>9.5</td>
<td>2.9</td>
<td>3.1</td>
</tr>
<tr>
<td>2017</td>
<td>79.9</td>
<td>38.5</td>
<td>14.0</td>
<td>4.4</td>
<td>1.2</td>
</tr>
<tr>
<td>2100</td>
<td>86.2</td>
<td>50.7</td>
<td>20.9</td>
<td>7.0</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Model predictions for 21st century. Wealth Shares in %.

- long-run effects of decrease in tax progressivity
Conclusion

Main findings:
- declining tax progressivity an important force for generating increases in wealth inequality (mostly a mechanical effect at the high end)
- speed of changes at the very top hard to match (if you believe in Saez & Zucman data)
- asset price movements with heterogeneous portfolios and/or time-varying inflation can account for short-run dynamics (U-shape in wealth inequality)

Questions:
- what accounts for speed of changes at very top?
- why are portfolios heterogeneous (both across and within wealth levels)?
- why are asset prices moving that much?
Perfect foresight vs. myopic transition

**top 10% wealth share**

- **Perfect foresight**
- **Myopic**

**top 1% wealth share**

**top 0.1% wealth share**

**top 0.01% wealth share**
 CES with elasticity of substitution $> 1$

$\sigma = 1.25$ (Karabarbounis and Neiman, 2014)
Decomposition: top shares I

Top 10%

Top 1%

Top 0.1%

Top 0.01%
How do savings vary with the (after-tax) interest rate for different types of consumers?

\( \omega \) is cash-on-hand, \( e \) is after-tax earnings, \( R \) is the gross effective after-tax interest rate, \( s \) is savings, \( u \) is CRRA with coefficient \( \sigma \):

\[
s^* = \arg \max_s [u(\omega - s) + \beta u(Rs + e)]
\]

Comparative statics:

\[
\frac{\partial s^*}{\partial R} \propto \omega(1 - \sigma) + \frac{e}{R} (\sigma + 1).
\]

If \( \sigma > 1 \), sign depends on relative sizes of \( \omega \) and \( e/R \).
s is savings, e is earnings, τ is the (flat) tax rate, i is the nominal interest rate, π is the inflation rate, and 
\[ r = \frac{(1 + i)}{(1 + \pi)} - 1 \] is the real rate.

Real cash-on-hand tomorrow is:
\[
\frac{s + (1 - \tau)(e(1 + \pi) + si)}{1 + \pi} = (1 - \tau)e + s \left( r(1 - \tau) - \tau \frac{\pi}{1 + \pi} \right) \approx (1 - \tau)e + s(r(1 - \tau) - \tau \pi).
\]

In other words, the real after-tax return \( \tilde{r} \) is given by
\[
\tilde{r} \approx r(1 - \tau) - \tau \pi
\]
Only changes in earnings risk I

- Top 10% wealth share
- Top 1% wealth share
- Top 0.1% wealth share
- Top 0.01% wealth share

Model vs. data (SZ) vs. data (SCF) over the years 1970 to 2010.
Only changes in earnings risk II
Only changes in top earnings shares
Only changes in top earnings shares II

**Capital - Net Output Ratio**
- Model (capital)
- Data (national wealth)
- Data (private wealth)

**Bottom 50% Share**
- Model
- Data (SCF)
Only changes in taxes I

**top 10% wealth share**

- Model (blue)
- Data (SZ, orange)
- Data (SCF, yellow)

**top 1% wealth share**

**top 0.1% wealth share**

**top 0.01% wealth share**
Only changes in taxes II

- Capital - net output ratio
  - Model (capital)
  - Data (national wealth)
  - Data (private wealth)

- Bottom 50% share
  - Model
  - Data (SCF)