Clearing, transparency, and collateral

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Abstract

In an environment of Over-The-Counter trading with asymmetric information about the quality of the counterparty, we study traders’ incentives to screen their counterparties under different clearing arrangements. When traders can choose the clearing arrangement, they decide which types of transactions to clear under each arrangement, with significant consequences for transparency and collateral requirements.

The key trade-off is between insurance and the value of information: on one hand risk averse traders want to smooth consumption and on the other hand they want to extract the largest feasible surplus from their counterparties. Choosing an arrangement that provides insurance, however, may prevent them from taking advantage of the private information they learn about their counterparty.

As a result, insurance comes at the cost of losing transparency: when clearing arrangements differ in the degree of risk sharing they implement, then they also differ in the degree of transparency arising in equilibrium. This has significant consequences on the role of collateral. In environments with limited commitment, collateral plays two roles for risk averse traders: it is a means to discipline incentives and to self-insure at the same time. When insurance is provided by a clearing arrangement that implements risk sharing, collateral is primarily used to discipline incentives, since risk sharing comes at the cost of less information about the limited commitment of the counterparty. The opposite is true in the equilibrium with a clearing arrangement that cannot provide risk sharing but gives traders sufficient incentives to screen their counterparties.

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1 Introduction

Relationships between traders are a key feature of financial markets: financial institutions can learn valuable information from multiple interactions with each other. In this paper we highlight a novel channel through which the information content of a financial transaction is affected: the clearing arrangement that is chosen for that transaction. Specifically, we study the consequences that different clearing arrangements have on the flow of information—which we refer to as transparency—to the clearing agent, and on the collateral required to insure against counterparty risk.

Modern financial institutions trade a variety of products bilaterally, such as over-the-counter (OTC) derivatives, repurchase agreements and even reserves at the central bank. Some of these markets are concentrated, with a few institutions holding a very large share of the total volume of trades on their balance sheets, and others are strongly connected, with most institutions having few counterparties. Thus, relationships may be very important to counterparties in a financial transaction as unique information can be acquired, for example, by multiple interactions with a given institution.

The recent financial crisis has highlighted the importance of these relationships and of the flow of information that they may carry through financial markets and different clearing arrangements.

Both academic research and policy makers have argued that the lack of transparency in over-the-counter markets may have led to uncertainty in traders’ positions during the crisis, which induced market participants to pull away from certain institutions and exacerbated their financial stress. In recent work Acharya and Bisin argue that central counterparty clearing has the potential to improve the degree of transparency in financial markets: the clearing entity interposes itself between the two parties to a transaction, becoming the buyer to every seller and the seller to every buyer. Therefore the clearing agent can observe all products traded by all the institutions for which it performs clearing, and can then offer individual institutions’ pricing schedules for trades that are contingent not just on publicly observable characteristics but also on its own knowledge of other trades. Also, because the central counterparty (CCP) makes the payments on behalf of the original trading counterparties, rather than trying to assess its exposure to all of its trading partners, a market participant needs to manage only its exposure to the central counterparty. And, through its better knowledge of the market, a CCP can impose more effective risk controls on clearing members. Thus counterparty risk can be appropriately insured.

Other work has suggested that costs arising from asymmetric information may instead be higher in centrally cleared markets than bilateral ones,
especially for complex products traded by complex, opaque intermediaries.\footnote{Dealers in bilateral markets specialize in valuing complex derivatives contracts and have opaque and risky balance sheets. Thus they are more effective than a CCP at monitoring and pricing the counterparty risk of other dealers ([19], pg. 4).}

Our paper formalizes this idea and studies a mechanism, different from that in [1], through which central counterparty clearing can affect transparency in over-the-counter markets, resulting in the opposite effect. In our model, financial transactions are bilateral and subject to limited commitment and private information: trading partners can pay a cost to learn the severity of the commitment problem of their counterparty—their type—and can then condition the contract on that information. An agent chooses to learn his counterparty’s type if the expected surplus he can extract from trading with such information exceeds its cost.

Also, because the product traded is subject to a shock, realized only after the contract is agreed upon, then a risk averse agent seeking to buy that product would also like to insure against the shock. When clearing is bilateral no such insurance is feasible. When clearing is done through a CCP, shocks that are idiosyncratic to a bilateral trade can be insured by mutualizing losses.\footnote{In practice this is done through a default or guarantee fund.}

Although it can provide risk pooling, the CCP may not have the same relationship with its members as bilateral counterparties have with each other. In particular, in our model the CCP cannot learn the severity of the commitment problem of the trading partners for which it does clearing. Thus, it has to elicit that information directly from the contract that the counterparties submit for clearing, and sets collateral requirements (i.e. margin) accordingly.

The key trade-off is then faced by trading partners with respect to their terms of trade: if they pay the cost to learn their counterparty’s type and choose to clear the transaction centrally, then loss mutualization implies that they will not be able to extract the surplus that their trade generates by taking advantage of the private information they have learned. On the other hand, they will be insured against the idiosyncratic shock to the product they are about to purchase. As a result, when the value of insurance against the idiosyncratic shock is large then agents choose not to learn their counterparty’s type and to clear the transaction centrally. In equilibrium collateral is then primarily used to discipline the limited commitment friction. When instead the value of idiosyncratic risk insurance is small relative to the expected gains from information (net of the cost of acquiring it) then agents choose to pay the cost to learn their counterparty’s type. In equilibrium, if the counterparty turns out to be good then they choose to clear bilaterally: collateral is then used as insurance against the idiosyncratic shock. If the counterparty turns out to be bad then they have nothing to lose from clearing centrally, so they choose to do so. The loss mutualization implemented by the CCP will in fact provide them with insurance against the idiosyncratic shock without any forgone gains from having to give up a good counterparty.

We also show that whenever central counterparty clearing is a feasible option for over-the-counter trades, transactions involving the riskiest assets are submit-
ted to the central counterparty for clearing. On the other hand, the riskier the counterparty is ex-ante, the higher the value of information about its creditworthiness is. Thus, for a given distribution of asset returns, a riskier counterparty leads to stronger incentives for traders to acquire information about that counterparty and clear bilaterally to extract the value of such information.

Notice that our results on transparency and collateral are robust to different trading assumptions. Suppose that, rather than trading bilaterally, agents contract directly with a pool of counterparties, so to diversify the idiosyncratic shock. Then, as long as the screening can be done only by one agent and the information does not become publicly known, agents always have an incentive not to screen. Thus no equilibrium with risk sharing and screening exists.

1.1 Related Literature

Our paper relates to a recent literature that studies how changes in financial market infrastructure influence the exposure to default risk, as well as market liquidity. Part of this literature has focused on the emergence of Central CounterParties as efficient institutions that help achieving Pareto superior allocations. Within this literature, [15] focus on counterparty risk insurance as the key mechanism that affects welfare: central counterparty clearing improves on bilateral clearing by providing market participants with better insurance against counterparty default. Such insurance is provided by a mechanism that uses novation and mutualization of losses, therefore resembling a CCP. [1] stress the welfare enhancing effect of central clearing on transparency. The key friction in their paper is the non observability of the trading positions that a counterparty in a financial transaction has with third parties. When the exposure to third parties can cause a counterparty to default, any trader who cannot observe this exposure will fail in correctly accounting for the counterparty default risk. Thus the non observability of trading positions imposes an externality that cannot be captured by competitive prices. The introduction of a CCP can correct this externality: by observing all products traded by all the institutions for which it performs clearing, the CCP can offer prices that convey all the relevant information. Thus the equilibrium is efficient. [9] focus on the provision of efficient clearing services: by clearing through a CCP market participants benefit from the multilateral netting of financial contracts with the CCP members. However, by clearing through a CCP, market participants lose from not being able to bilaterally net different financial contracts with the same bilateral counterparty. If the former effect is stronger than the latter, then a CCP is welfare improving. [18] focus on two-sided limited commitment, and show that the introduction of a technology that allows to pledge collateral at a third party, a segregation technology, improves welfare with respect to bilateral clearing, and that a CCP is able to offer additional added-value through novation and mutualization.

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The only exception being, in our model, the case where the value of idiosyncratic risk insurance is small and agents learn that they face a bad counterparty, so that they prefer to clear centrally and be insured. However, given an arrangement that provide risk sharing, rather than agents being able to choose it, screening never occurs.
nally, \cite{8} highlight a liquidity enhancing role for CCPs: by providing services that are valuable to market participants when they trade, CCPs reduce the (opportunity) costs related to trading. Thus they facilitate socially desirable transactions that would otherwise not occur (due to adverse selection arising endogenously in their model).

We focus on novation and mutualization of losses as key features of central clearing, and how they affect traders’ incentives to learn information—that is socially valuable—about their trading partners.

In this respect, our paper is closer to \cite{14} and \cite{5}, who study to what extent clearing can improve on the allocation of risk, and whether clearing should be bilateral, decentralized, or centralized. \cite{14} considers an environment with moral hazard in which, depending on the severity of the moral hazard problem, market participants can use collateral either as an incentive device, or an insurance device. When a CCP cannot observe the degree of moral hazard, it can only use collateral either as an incentive or as an insurance device. If the CCP opts to use collateral as an incentive device, by preventing insurance contracts to take place, central clearing can have the unintended consequence of forcing collateral to increase for all contracts.

\cite{5} study an environment with idiosyncratic risk, aggregate risk, and moral hazard. Specifically, in their paper, risk averse protection buyers own an asset that is subject to an idiosyncratic and to an aggregate shock. To cope with these shocks buyers can purchase insurance from risk neutral protection sellers; the problem is that protection sellers themselves may default. In this context, a clearing entity can insure protection buyers against the default of their counterparty, and when clearing is centralized a central clearing entity, a CCP, can offer insurance to all protection buyers. The benefit of central clearing is that it allows buyers to diversify their idiosyncratic risk: protection buyers can pool risk among themselves. Nevertheless, central clearing cannot provide insurance against aggregate risk, and sellers come into play. Because protection sellers differ in their probability of defaulting, and protection buyers can exert effort to find relatively safer protection sellers, when aggregate risk is significant, it is efficient to have buyers exerting effort to find safe counterparties. Nevertheless, when such effort is privately costly, incentive compatibility requires the CCP to not fully insure protection buyers against their idiosyncratic risk.

2 The Model

Time is discrete and consists of two periods, \( t = 1, 2 \). The economy is populated by two type of agents: a measure one of lenders and a measure one of borrowers, who have different preferences and have access to different technologies.

There are two goods, a consumption good and a capital good. The capital good can be invested in time \( t = 1 \) and transformed, via a linear technology, into time \( t = 2 \) consumption good. Nevertheless, only borrowers have access to such a technology. The technology is indivisible, takes one unit of capital good in \( t = 1 \), and returns \( \theta \) units of consumption good in \( t = 2 \). The realization \( \theta \) is
a random variable, with support \( \{0, \theta\} \) and \( p = \text{Prob}(\tilde{\theta} = \theta) \), and it is unknown when the investment is undertaken.

The consumption good can be stored from \( t = 1 \) to \( t = 2 \) both by lenders and borrowers, but the latter enjoy more consumption in the first period. Specifically, a borrower preferences are defined over time \( t = 1 \) consumption \( c_1 \) and time \( t = 2 \) consumption \( c_2 \), and are represented by the utility function

\[ U(c_1, c_2) = \alpha c_1 + c_2 \quad \alpha > 1 \]

In the first period, lenders receive an endowment of one unit of capital, while borrowers receive an endowment of \( \omega \) units of consumption good.

In this economy resources are in the wrong hands: lenders have capital but they need borrowers to use their technology to transform it into consumption goods. This creates incentives from trade. Nevertheless, trade is subject to two frictions. First, each lender is randomly matched and can only contract with one borrower: trade is bilateral. Second, borrowers have limited commitment to repay: a borrower can repudiate a contract and, after default, get away with a fraction \( 1 - \lambda_i \) of the output realization. We assume that there are two types of borrowers: a measure \( q \) of good borrowers, indexed by a parameter \( \lambda_H \), and a measure \( 1 - q \) of bad borrowers, indexed by the parameter \( \lambda_L \), where \( \lambda_H > \lambda_L \).

The type \( \lambda_i \) is private information of the borrower, but can be learned by a lender by investing some resources into monitoring effort. Specifically, the preferences of a lender are defined only over second period consumption \( x_2 \), and time 1 monitoring effort \( e_1 \), according with the utility function

\[ V(x_2, e_1) = u(x_2) - \gamma \cdot e_1 \]

where \( u \) is strictly increasing and strictly concave function, and \( e_1 \in \{0, 1\} \). We further assume that \( \lim_{x \to 0} u(x) = \infty \).

Contracts are negotiated bilaterally by a lender and a borrower in each match, but they can be settled bilaterally or centrally (via a Central counter-Party) in the second period. In the next sections, we take the market structure as given, and then we compare the properties of the solutions with central and bilateral clearing.

3 Bilateral clearing

After lenders and borrowers are randomly matched at the beginning of \( t = 1 \), a lender chooses whether to verify the counterparty type, \( e_1 = 1 \), or not, \( e_1 = 0 \).

With bilateral clearing, if the lender verifies the counterparty’s type, he offers a contract \((x_{2,1}^i, x_{2,1}^j, c_{1,1}^i, c_{1,1}^j, c_{2,1}^i, c_{2,1}^j)\), where \( x_{1,s}^i \) and \( c_{1,s}^i \) are respectively the lender and the borrower consumption in time \( t \) and state \( s \), when the match is between a lender and a type \( i \) borrower. A contract is then generally indexed by the borrower’s type \( i \), and second period consumption is indexed by idiosyncratic

\[ \text{Since } \lambda_H > \lambda_L, \text{ borrowers with } \lambda_H \text{ can abscond a lower fraction of output. In this sense, } \lambda_H \text{ borrowers are good borrowers.} \]
state $s \in \{h, l\}$. If the borrower accepts the contract, (i) the lender transfers the one unit of capital to the borrower, (ii) the borrower transfers the lender the consumption good $\omega$ and (iii) receives in exchange consumption $c^i_1$. We can think of the difference $\omega - c^i_1$ as collateral. The borrower then invests the one unit of capital in his technology, while the lender stores the consumption good $\omega - c^i_1$. In the second period, the borrower transfers $\tilde{\theta}$ to the lender and the two consume $c^i_{2,s}$ (the borrower) and $x^i_{2,s}$ (the lender).

If the lender does not verify the counterparty type, he offers a menu of contracts $(x^i_{2,h}, x^i_{2,l}, c^i_1, c^i_{2,h}, c^i_{2,l})$, where each type self-selects into type $i$ contract. Then, the borrower transfers the consumption good $\omega$, and he receives the one unit of capital and consumption $\omega - c^i_1$. In the second period, after the shock realizes, the borrower transfers the realized investment $\tilde{\theta}$, as long as $c^i_{2,s} \geq (1 - \lambda_i)\theta$. If the borrower transfers the consumption good, the lender consumes $\lambda^i\theta$, where $\lambda^i$ is the borrower pledgeability of assets.

We characterize first the contract where the lender pays the cost $\gamma$ and acquires information about his borrower’s type and then analyze contracts without information acquisition. Our focus is on the former contract: we are interested in economies where the equilibrium under bilateral clearing is such that lenders choose to costly learn the quality of their counterparty. When characterizing the contract with central clearing we will focus mostly on the contract where the CCP pools the borrowers’ types over the lowest type in the support, $\lambda^L$ and characterize equilibria with a contract that allows separation of types or pooling over $\lambda^H$ later on.

Our goal is to show that there exist economies where the equilibrium with bilateral clearing features information acquisition but the equilibrium with central clearing features pooling over $\lambda^L$ and thus no information is ever acquired. Because this information is specific to the bilateral relationship, it can only be acquired by lenders: thus if in equilibrium central clearing is chosen then this information is lost. However we will show that such loss of information is efficient: the reason is that no externalities exist in this economy. Information is not useful to anything but the bilateral relationship, therefore when lenders prefer clearing through a CCP that is also what a social planner prefers.

### 3.1 The contract with information acquisition

Lenders matched with a $\lambda^H$ borrower solve a similar problem to the one that lenders matched with $\lambda^L$ borrowers face, with the only difference of the limited commitment problem.

Let $V_i$ denote the value to a lender of a match with borrower of type $\lambda_i$, once the lender paid the cost $\gamma$ and knows the type of the borrower he’s matched with. Then lenders choose contracts $(x^{i}_{2,h}, x^{i}_{2,l}, c^{i}_1, c^{i}_{2,h}, c^{i}_{2,l})$ to solve

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12Storage is verifiable.
\[ V_i = \max_{(x_{2,h}, x_{2,l}, c_i^1, c_i^2, h, c_i^2, l) \in \mathbb{R}_+^5} \] 
\[ p u(x_{2,h}) + (1 - p) u(x_{2,l}) - \gamma \] 
\[ s.t. \quad \alpha c_i^1 + p c_i^2, h + (1 - p) c_i^2, l \geq \alpha \omega \] 
\[ c_i^1 \leq \omega \] 
\[ c_i^2, h + x_{2,h} \leq \omega - c_i + \theta \] 
\[ c_i^2, l + x_{2,l} \leq w - c_i \] 
\[ c_i^2, h \geq (1 - \lambda_i) \theta \] 

Constraint (2) is the borrower’s participation constraint: the borrower can always refuse to trade, and consume the endowment \( \omega \) in the first period. Constraint (3) is time one feasibility, (4) and (5) are time two feasibility in states \( h \) and \( l \) respectively. Constraint (6) is the borrower individual rationality constraint: the borrower can default and run away with \( 1 - \lambda_i \) of the consumption good (in the low state \( \theta = 0 \) the limited commitment to repay creates no problems).

Notice that when \( \alpha > 1 \) the only reason for the lender to pay the borrower with less resources at \( t = 1 \) and more at \( t = 2 \) in the state with high output realization (i.e. limited commitment constraint is slack) is that storing resources from \( t = 1 \) to \( t = 2 \) is the only way for the lender to guarantee more consumption goods in the state with low output realization.

It is easy to see that at a solution it must be that (4) and (5) are binding:

\[ c_i^2, h + x_{2,h} = \omega - c_i^1 + \theta \] 
\[ c_i^2, l + x_{2,l} = \omega - c_i^1 \] 

**Lemma 1** When the lender pays the cost \( \gamma \) to screen the borrower, and clearing is bilateral, collateral is always positive, meaning \( c_i^1 < \omega \). Moreover, because collateral is costly, \( c_i^2, l = 0 \) and \( x_{2,h} > x_{2,l} \).

Counterparty risk is managed over time through clearing: the borrower may be unable or unwilling to settle the underlying position. Colateral margins, in the amount \( \omega - c_i^1 \), reduce this risk. Nevertheless collateral must be used efficiently, because it is costly. Therefore \( c_i^2, l = 0 \) and insurance is incomplete.

**Lemma 2** Given \( \lambda^i \), if \( \omega < \frac{(1-\lambda^i)p\theta}{\alpha} \), then the limited commitment constraint is binding, the participation constraint is slack and \( c_i = 0 \). This is area 4 in figure.

**Proof.** Suppose that \( \omega < \frac{(1-\lambda^i)p\theta}{\alpha} \). Because \( c_i^2, h \geq (1 - \lambda^i) \theta \), we have easily that the participation constraint is slack, since \( c_i^1 \geq 0 \). It follows than that \( c_i^1 = 0 \) and \( c_i^2, h = (1 - \lambda^i) \theta \).
Collateral is always in place to provide incentives to repay. When collateral is scarce, meaning $\omega < \frac{(1-\lambda^i)p\theta}{\alpha}$, the borrower is asked to post all of his collateral in $t = 1$.

Consider now the case of $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$. After replacing the values of $x^i_{2,h}$, $x^i_{2,l}$, and $c^i_{2,l}$, let $\mu$ and $\eta$ be the multipliers associated with (82) and (83) respectively. The first order conditions for optimality are

$$-pu'(\omega - c^i_1 + \theta - c^i_{2,h}) + p\mu + \eta = 0$$  \hspace{1cm} (7)$$

$$-pu'(\omega - c^i_1 + \theta - c^i_{2,h}) - (1-p)u'((\omega - c^i_1) + \alpha \mu \leq 0$$  \hspace{1cm} (8)$$

with equality if $c^i_1 > 0$. Together with the complementary slackness conditions

$$\mu \{\alpha c^i_1 + p c^i_{2,h} - \alpha \omega\} = 0$$  \hspace{1cm} (9)$$

and

$$\eta \{c^i_{2,h} - (1-\lambda^i)\theta\} = 0$$  \hspace{1cm} (10)$$

fully characterize the solution to the problem. Let $\lambda^*$ be the unique value satisfying

$$\frac{\alpha - p}{1 - p} = \frac{u' \left( \frac{(1-\lambda^*)p\theta}{\alpha} \right)}{u' \left( \theta - \frac{\alpha - p}{p} (1-\lambda^*)\theta \right)}$$

**Lemma 3** When the lender pays the cost $\gamma$ to screen the borrower, clearing is bilateral, and $\omega > \frac{(1-\lambda^i)p\theta}{\alpha}$, the participation constraint (2) binds. Moreover,

a If $\lambda < \lambda^*$, then $c^i_{2,h} = (1-\lambda^i)\theta$ and $c_1 = \omega - \frac{(1-\lambda)p\theta}{\alpha}$,

b If $\lambda > \lambda^*$, and $\omega < \frac{(1-\lambda^*)p\theta}{\alpha}$, then $c^i_{2,h} > (1-\lambda^i)\theta$ and $c^i_1 = 0$.

c If $\lambda > \lambda^*$, and $\omega \geq \frac{(1-\lambda^*)p\theta}{\alpha}$, then $c^i_{2,h} = \alpha \frac{\omega - c^i_1}{p} > (1-\lambda^i)\theta$, for $c^i_1$ solving

$$\frac{\alpha - p}{1 - p} = \frac{u' \left( \omega - c^i_1 \right)}{u' \left( \theta - \frac{\alpha - p}{p} (\omega - c^i_1) \right)}$$

The solution to the problem is shown in Figure 1. The partition of the state space depends on two key parameters: the collateral endowment $\omega$ and the value of the borrower $\lambda^i$ (or the borrower’s temptation to default $1 - \lambda^i$). The interaction of the two determines whether both the limited commitment and the participation constraint bind or only one of them binds.

In the bilateral problem $\omega$ plays a double role: it is the collateral needed for insurance purposes (against the high or low realization of $\tilde{\theta}$) and it is also a lower bound on the payoff the borrower needs to participate in the contract. The temptation to default $1 - \lambda^i$ measures the severity of the commitment problem so that when $\lambda^i$ is relatively low the borrower is relatively bad, then the solution
to \(\omega\) must be such that the limited commitment constraint binds. Then we can distinguish two scenarios: when \(\omega\) is relatively low \((\omega \leq (1-\lambda)p\theta\alpha)\) then it is relatively easy to get the borrower to participate in the contract, that is to say the participation constraint is slack. Then the limited commitment constraint binds. On the other hand, because \(\omega\) is relatively scarce, then it is valuable to provide insurance to the lender: thus \(c_1 = 0\). This is solution 1 and area 4 in figure 1.

When \(\omega\) is relatively high the participation constraint starts binding: this is true for solutions 2a, 2b, 2c and areas 1, 2, 3 in figure 1. Whether the limited commitment constraint binds \(\text{(PC)}\) or not \(\text{(LC)}\) depends on the severity of the commitment problem with respect to a threshold \(\lambda^*\): if \(\lambda \leq \lambda^*\) then the borrower’s temptation to default is high, so the limited commitment constraint binds. If \(\lambda > \lambda^*\) then the borrower’s temptation to default is low, so the limited commitment constraint is slack. In this last case, however, we can distinguish two subcases: the first where the borrower’s endowment is more valuable as collateral rather than as an instrument to provide incentives \((\omega \leq (1-\lambda^*)p\theta\alpha)\) so that in equilibrium \(c_1 = 0\). The second where the borrower’s endowment is relatively abundant and is more valuable to provide incentives \((\omega > (1-\lambda^*)p\theta\alpha)\) so that in equilibrium \(c_1 > 0\). Because the commitment problem is not severe (the limited commitment constraint is slack) in these two cases, then the relevant threshold for the borrower’s endowment is a function of \(\lambda^*\) rather than the actual temptation to default in the bilateral trade.

![Figure 1: Solution to bilateral problem.](image)

\(^{13}\)Solution 2a and area 3 in figure 1

\(^{14}\)Solutions 2b, 2c and areas 1, 2 in figure 1
3.2 Bilateral contract without information acquisition

Let $V^{bil,\lambda H}$, $V^{bil,\lambda L}$ denote the pooling contracts that a lender who is clearing the contract bilaterally gets if only the limited commitment constraint of the $\lambda^H$-borrower is satisfied or the limited commitment constraint of the $\lambda^L$-borrower is satisfied (and hence both are). Thus:

$$V^{bil,\lambda H} = \max \{ p \left[ q u (x_{2h}^H) + (1 - q) u \left( \min \{ x_{2h}^L, x_{2h}^H \} \right) \right] + (1 - p) u (x_{2l}) \}$$

s.t. $c_1 \leq \omega$

$$\alpha c_1 + p c_{2h} + (1 - p) c_{2l} \geq \alpha \omega$$

$$c_{2h} \geq (1 - \lambda^H) \theta$$

$$x_{2h}^H + c_{2h} \leq \omega - c_1 + \theta$$

$$x_{2l}^H + (1 - \lambda^H) \theta \leq \omega - c_1 + \theta$$

$$x_{2l} + c_{2l} \leq \omega - c_1$$

and

$$V^{bil,\lambda L} = \max \{ p u (x_{2h}) + (1 - p) u (x_{2l}) \}$$

s.t. $c_1 \leq \omega$

$$\alpha c_1 + p c_{2h} + (1 - p) c_{2l} \geq \alpha \omega$$

$$c_{2h} \geq (1 - \lambda^L) \theta$$

$$x_{2h}^H + c_{2h} \leq \omega - c_1 + \theta$$

$$x_{2l} + c_{2l} \leq \omega - c_1$$

For both problems we have that $c_{2l} = 0$ and the $t = 2$ resource constraint bind.

For the first problem, yielding $V^{bil,\lambda H}$, we have that FOC for $c_1$ and $c_{2h}$ are:

$$p \left[ -q u' (\omega - c_1 + \theta - c_{2h}) - (1 - q) u' (\omega - c_1 + \min (\theta - c_{2h}, \lambda^L \theta)) \right] - (1 - p) u' (\omega - c_1) + \alpha \mu \leq 0$$

$$p \left[ -q u' (\omega - c_1 + \theta - c_{2h}) - \mathbf{1}_{c_{2h} > (1 - \lambda^L) \theta} (1 - q) u' (\omega - c_1 + \theta - c_{2h}) \right] + \mu p + \eta = 0$$

where $\mu, \eta$ denote the Lagrange multipliers on the PC and LC respectively.

Thus a solution is such that:

1. Either both constraints bind: in this case $c_{2h} = (1 - \lambda^H) \theta < (1 - \lambda^L) \theta$, and $c_1 = \omega - \frac{p(1-\lambda^H)\theta}{\alpha}$. So that $x_{2l} = \frac{p(1-\lambda^H)\theta}{\alpha}$, $x_{2h} = \frac{p(1-\lambda^H)\theta}{\alpha} + \lambda^H \theta$,

$$x_{2h}^L = \frac{p(1-\lambda^H)\theta}{\alpha} + \lambda^L \theta.$$

So:

$$V^{bil,\lambda H} = p \left[ q u \left( \frac{p(1-\lambda^H)\theta}{\alpha} + \lambda^H \theta \right) \right] + (1 - q) u \left( \frac{p(1-\lambda^H)\theta}{\alpha} + \lambda^L \theta \right)$$


11
(α - p) qu' \left( \frac{p (1 - \lambda H \theta)}{\alpha} + \lambda H \theta \right) > p (1 - q) u' \left( \frac{p (1 - \lambda H \theta)}{\alpha} + \lambda L \theta \right) + (1 - p) u' \left( \frac{p (1 - \lambda H \theta)}{\alpha} \right)

2. or only one constraint binds:

(a) LC binds and PC is slack: \(c_{2h} = (1 - \lambda H) \theta < (1 - \lambda L) \theta\) and \(c_1 = 0\).

This is a solution if and only if \(\omega < \frac{p(1-\lambda L)\theta}{\alpha}\). Then:

\[ V^{bi},\lambda H = p \left[ qu \left( \omega + \lambda H \theta \right) + (1 - q) u \left( \omega + \lambda L \theta \right) \right] + (1 - p) u (\omega) \]

(b) PC binds and LC is slack:

\[ p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) \right] + (1 - p) u' (\omega - c_1) \]

\[ \alpha \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + 1_{c_{2h} > (1 - \lambda L) \theta} (1 - q) u' \left( \omega - c_1 + \theta - c_{2h} \right) \right] \leq p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) \right] + (1 - p) u' (\omega - c_1) \]

i. \(c_{2h} = \frac{\omega}{p} < (1 - \lambda L) \theta, c_1 = 0\) then \(x_{2l} = \omega, x_{2h}^H = \omega + \theta - \frac{\omega}{p}, x_{2h}^L = \omega + \lambda L \theta\). So:

\[ V^{bi},\lambda H = p \left[ qu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - q) u \left( \omega + \lambda L \theta \right) \right] + (1 - p) u (\omega) \]

This is a solution if and only if \(\omega > \frac{p(1-\lambda H)\theta}{\alpha}\) and, for \(c_1 = 0\):

\[(\alpha - p) qu' \left( \omega - c_1 + \theta - c_{2h} \right) < p (1 - q) u' \left( \omega - c_1 + \min \left( \theta - c_{2h}, \lambda L \theta \right) \right) + \]

\[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) > p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) \right] + (1 - p) u' (\omega - c_1) \]

\[(\alpha - p) qu' \left( \omega - c_1 + \theta - c_{2h} \right) > p (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) + (1 - p) u' (\omega - c_1) \]

\[\text{Combining the FOC for } c_1 \text{ and } c_{2h} \text{ we have that both constraints bind if and only if:}\]

\[ \alpha u = p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \min \left( \theta - c_{2h}, \lambda L \theta \right) \right) \right] + (1 - p) u' (\omega - c_1) \]

\[ = p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) \right] + (1 - p) u' (\omega - c_1) \]

\[ \mu p + \eta = p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + 1_{c_{2h} > (1 - \lambda L) \theta} (1 - q) u' \left( \omega - c_1 + \theta - c_{2h} \right) \right] \]

\[ = pqu' \left( \omega - c_1 + \theta - c_{2h} \right) \]

so that:

\[ \alpha qu' \left( \omega - c_1 + \theta - c_{2h} \right) > p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) \right] + (1 - p) u' (\omega - c_1) \]

\[ (\alpha - p) qu' \left( \omega - c_1 + \theta - c_{2h} \right) > p (1 - q) u' \left( \omega - c_1 + \lambda L \theta \right) + (1 - p) u' (\omega - c_1) \]
\[(1-p)u'(\omega-c_1)\]

which, evaluated at this solution, boils down to:

\[(\alpha-p)qu'\left(\omega\left(1-\frac{\alpha}{p}\right)+\theta\right) < p(1-q)u'(\omega+\lambda^L\theta)+(1-p)u'(\omega)\]

ii. \(c_{2h} = \frac{\alpha\omega}{\omega + \theta - \frac{\alpha\omega}{p}}\). So:

\[V_{bil,\lambda H}^{bil} = pu\left(\omega + \theta - \frac{\alpha\omega}{p}\right) + (1-p)u(\omega)\]

This is a solution if and only if \(\omega \geq \frac{p(1-\lambda^H)\theta}{\alpha}\) and, for \(c_1 = 0\):

\[(\alpha-p)u'\left(\omega\left(1-\frac{\alpha}{p}\right)+\theta\right) < (1-p)u'(\omega)\]

iii. \(c_{2h} = \alpha(\omega-c_{1^*})^\frac{1}{p} < (1-\lambda^L)\theta; c_1 = c_{1^*} > 0\) then \(x_{2l} = \omega - c_{1^*}\), \(x_{2h}^L = \omega - c_{1^*} + \lambda^L\theta\). So:

\[V_{bil,\lambda H}^{bil} = p\left[qu\left(\omega-c_{1^*} + \theta - \frac{\alpha(\omega-c_{1^*})}{p}\right) + (1-q)u(\omega-c_{1^*} + \theta - (1-\lambda^L)\theta)\right] + (1-p)u(\omega-c_{1^*})\]

This is a solution if and only if \(\omega \geq \frac{p(1-\lambda^H)\theta}{\alpha}\) and \(c_{1^*}\) solves \(\boxed{16}\)

\[(\alpha-p)qu'\left(\omega-c_1 + \theta - \frac{\alpha(\omega-c_{1^*})}{p}\right) = p(1-q)u'(\omega-c_1 + \lambda^L\theta) + (1-p)u'(\omega-c_1).

Claim 4 \(c_{1^*} < c_1^*\).

**Proof.** Rewrite the condition for optimality of \(c_1^*\):

\[(\alpha-p)u'\left(\theta - \frac{\alpha-p}{p}[\omega - c_{1^*}]\right) = (1-p)u'(\omega - c_{1^*})\]

and for \(c_{1^*}^*\):

\[(\alpha-p)qu'\left(\theta - \frac{\alpha-p}{p}[\omega - c_{1^*}^*]\right) = (1-p)u'(\omega - c_{1^*}) + p(1-q)u'(\omega - c_{1^*} + \lambda^L\theta)\]

If we plot the LHS and the RHS of the two equations as a function of \(c_1\), we find easily that \(c_{1^*}^* > c_{1^*}^*\).  

---

16From the FOC for \(c_1\) and \(c_{2h}\) with \(\eta = 0\) since the LC constraint is slack, we have:

\[
\alpha qu' (\omega - c_1 + \theta - c_{2h}) = p \left[qu' (\omega - c_1 + \theta - c_{2h}) + (1-q)u' (\omega - c_1 + \lambda^L\theta)\right] + (1-p)u'(\omega - c_1)
\]

\[
(\alpha-p)qu' (\omega - c_1 + \theta - \frac{\alpha\omega-c_1}{p}) = p(1-q)u' (\omega - c_1 + \lambda^L\theta) + (1-p)u'(\omega - c_1)
\]
iv. \( c_{2h} = \frac{\alpha(\omega - c_1^*)}{p} > (1 - \lambda L) \theta \), \( c_1 = c_1^* > 0 \) then \( x_{2l} = \omega - c_1^* \), \\
\( x_{2h}^L = x_{2h}^H = \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \). So:

\[
V^{bil, \lambda H} = pu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - p) u \left( \omega - c_1^* \right)
\]

This is a solution if and only if \( \omega \geq \frac{p(1 - \lambda H)\theta}{\alpha} \) and \( c_1^* \) solves\(^{17}\)

\[
\frac{\alpha - p}{1 - p} = \frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p})}
\]

For the second problem, yielding \( V^{bil, \lambda L} \) we have that either both constraints bind, or only one does, and the solution is exactly the same as in the case of a lender who knows he is facing a \( \lambda^L \) borrower:

1. both LC and PC bind: \( c_{2h} = (1 - \lambda^L) \theta \), \( c_1 = \omega - \frac{(1 - \lambda^L)\theta}{\alpha} \) if and only if \( \omega \geq \frac{(1 - \lambda^L)\theta}{\alpha} \) and \( \frac{\alpha - p}{1 - p} > \frac{u' \left( \frac{(1 - \lambda^L)\theta}{\alpha} \right)}{u' \left( \lambda^L \theta + \frac{(1 - \lambda^L)\theta}{\alpha} \right)} \). Thus \( V^{bil, \lambda L} = pu \left( \frac{(1 - \lambda^L)\theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u \left( \frac{(1 - \lambda^L)\theta}{\alpha} \right) \)

2. LC binds and PC is slack: \( c_{2h} = (1 - \lambda^L) \theta \), \( c_1 = 0 \). This is a solution to the lender’s problem if and only if \( \omega < \frac{(1 - \lambda^L)\theta}{\alpha} \). Thus \( V^{bil, \lambda L} = pu \left( \omega + \lambda^L \theta \right) + (1 - p) u \left( \omega \right) \)

3. LC is slack and PC binds: \( c_1 = 0 \) or \( c_1 = c_1^* > 0 \). This is a solution to the lender’s problem if and only if \( \omega \geq \frac{(1 - \lambda^L)\theta}{\alpha} \). \( c_1 = 0 \) is part of a solution if and only if \( \frac{\alpha - p}{1 - p} < \frac{u' \left( \frac{(\omega - c_1^*)}{p} + \omega \right)}{u' \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right)} \). Thus

\[
V^{bil, \lambda L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u \left( \omega \right)
\]

and \( c_1 = c_1^* > 0 \) solving \( \frac{\alpha - p}{1 - p} = \frac{u' \left( \omega - c_1^* \right)}{u' \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right)} \) is part of a solution if

and only if \( c_{2h} = \frac{\alpha(\omega - c_1^*)}{p} > (1 - \lambda L) \theta \). Thus \( V^{bil, \lambda L} = pu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - p) u \left( \omega - c_1^* \right) \)

\(^{17}\)From the FOC for \( c_1 \) and \( c_{2h} \) with \( \eta = 0 \) since the LC constraint is slack, we have

\[
\begin{align*}
\alpha \left[ pu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \theta - c_{2h} \right) \right] &= \\
p \left[ qu' \left( \omega - c_1 + \theta - c_{2h} \right) + (1 - q) u' \left( \omega - c_1 + \theta - c_{2h} \right) \right] + (1 - p) u' \left( \omega - c_1 \right) &= 0
\end{align*}
\]
And notice that as in the case of the CCP, also in the bilateral problem it is not possible for the lender to elicit the borrower’s type if he does not acquire information. In fact, without information acquisition, the lender’s problem would be subject to a truth telling constraint similar to the one we had in the paper for the CCP:

\[
V_{bil} = \max \{ pu \left( X^i_{2h} \right) + (1 - p) u \left( X^i_1 \right) \}
\]

s.t. \[
\alpha C^i_1 + p C^i_{2h} + (1 - p) C^i_2 \geq \alpha \omega, \forall i
\]
\[
C^i_{2h} \geq (1 - \lambda^i) \theta, \forall i
\]
\[
C^i_1 \leq \omega
\]
\[
X^i_{2h} + C^i_{2h} \leq \omega - C^i_1 + \theta
\]
\[
X^i_{2l} + C^i_{2l} \leq \omega - C^i_1
\]
\[
\alpha C^i_1 + p C^i_{2h} + (1 - p) C^i_{2l} \geq \alpha C^{-i}_1 + p \max \left( C^{-i}_{2h}, (1 - \lambda^i) \theta \right) + (1 - p) C^{-i}_{2l}, \forall i
\]

As in the case for \(V_{bil,\lambda H}\), it is easy to show that at a solution \(C^i_{2l} = 0\), the resource constraint at \(t = 1\) is slack, while the resource constraint at \(t = 2\) in both states of the world bind, so that:

\[
V^{bil} = \max \{ pu \left( \omega - C^i_1 + \theta - C^i_{2h} \right) + (1 - p) u \left( \omega - C^i_1 \right) \}
\]

s.t. \[
\alpha C^i_1 + p C^i_{2h} \geq \alpha \omega, \forall i
\]
\[
C^i_{2h} \geq (1 - \lambda^i) \theta, \forall i
\]
\[
\alpha C^i_1 + p C^i_{2h} \geq \alpha C^{-i}_1 + p \max \left( C^{-i}_{2h}, (1 - \lambda^i) \theta \right), \forall i
\]

Clearly, it must be that \(C^i_{2h} \geq (1 - \lambda^i) \theta, \forall i\) to satisfy the LC constraint for both types. This implies that the truth-telling constraint is simply \(\alpha C^i_1 + p C^i_{2h} \geq \alpha C^{-i}_1 + p C^{-i}_{2h}, \forall i\) and therefore \(\alpha C^i_1 + p C^i_{2h} = \alpha C^{-i}_1 + p C^{-i}_{2h}\). So this problem is equivalent to \(V^{bil,\lambda L}\).

### 3.2.1 Optimal bilateral contract without information acquisition

When clearing bilaterally without acquiring information, the best contract depends on the region of the parameters’ space we are in:

1. if \(\omega < \frac{(1 - \lambda^H) p \theta}{\alpha} < \frac{p(1 - \lambda^L) \theta}{\alpha}\) then LC binds and PC is slack in both pooling contracts, and

   \[
   V^{bil,\lambda H} = p \left[ qu \left( \omega + \lambda^H \theta \right) + (1 - q) u \left( \omega + \lambda^H \theta \right) \right] + (1 - p) u \left( \omega \right)
   \]

   \[
   V^{bil,\lambda L} = pu \left( \omega + \lambda^L \theta \right) + (1 - p) u \left( \omega \right)
   \]

so it is obvious that the pooling contract over \(\lambda^H\) is chosen. This is because \(\omega\) is so low that \(c_1 = 0\) in both pooling contracts. If collateral (\(\omega\)) is abundant, the cost of offering a pooling over \(\lambda^H\) contract is that a \(\lambda^L\) type consumes more \(c_1\) than he should, and then defaults in the second
period. Since here $c_1 = 0$ in both contracts because collateral is scarce, then there is no cost in offering a $\lambda_H$ contract to a $\lambda_L$ type.

2. if \( \frac{(1-\lambda_H)\theta}{\alpha} < \omega < \frac{(1-\lambda_L)\theta}{\alpha} \) then LC binds and PC is slack in the pooling contract over $\lambda_L$, and for the pooling contract over $\lambda_H$:

(a) both LC and PC bind, which happens only if

\[
(\alpha - p) q u' \left( \frac{p (1 - \lambda_H) \theta}{\alpha} + \lambda_H \theta \right) > p (1 - q) u' \left( \frac{p (1 - \lambda_H) \theta}{\alpha} + \lambda_H \theta \right) + (1 - p) u' \left( \frac{p (1 - \lambda_H) \theta}{\alpha} \right)
\]

then

\[
V^{bil,\lambda_H} = p \left[ qu \left( \frac{p (1 - \lambda_H) \theta}{\alpha} + \lambda_H \theta \right) + (1 - q) u \left( \frac{p (1 - \lambda_H) \theta}{\alpha} + \lambda_L \theta \right) \right] + (1 - p) u \left( \frac{p (1 - \lambda_H) \theta}{\alpha} \right)
\]

\[
V^{bil,\lambda_L} = p u \left( \omega + \lambda_L \theta \right) + (1 - p) u (\omega)
\]

Here there is a trade off: if and only if $q$ is relatively small with respect to how risk averse lenders are, then the pooling contract over $\lambda_L$ is preferred.

(b) LC is slack and PC binds, and $c_1 = 0$ (which is Case 2.b.ii in the pooling contract over $\lambda_H$).

i. if $c_{2h} = \frac{\alpha \omega}{p} > (1 - \lambda_L) \theta$ only if $(\alpha - p) u' \left( \omega \left( 1 - \frac{\alpha}{p} \right) + \theta \right) < (1 - p) u' (\omega)$ then

\[
V^{bil,\lambda_H} = p u \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u (\omega)
\]

\[
V^{bil,\lambda_L} = p u \left( \omega + \lambda_L \theta \right) + (1 - p) u (\omega)
\]

so it is obvious that the pooling contract over $\lambda_L$ is chosen. However this case can never arise, because we assumed that we are in the region of the parameter space where $\omega < \frac{p (1 - \lambda_L) \theta}{\alpha}$.

ii. if $c_{2h} = \frac{\alpha \omega}{p} < (1 - \lambda_L) \theta$ and if $(\alpha - p) q u' \left( \omega \left( 1 - \frac{\alpha}{p} \right) + \theta \right) < p (1 - q) u' \left( \omega + \lambda_L \theta \right) + (1 - p) u' (\omega)$ then (Case 2b.i in the pooling over $\lambda_H$)

\[
V^{bil,\lambda_H} = p \left[ qu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - q) u \left( \omega + \lambda_L \theta \right) \right] + (1 - p) u (\omega)
\]

\[\text{This implies that information is never acquired in the bilateral contract, and therefore in the CCP is not acquired either. Because the CCP pools over $\theta$ shocks, CCP clearing is always preferred in this case.}\]
\[ V^{bil, \lambda L} = pu \left( \omega + \lambda^L \theta \right) + (1 - p) u \left( \omega \right) \]

Here the pooling contract over \( \lambda^H \) is always preferred, because \( \frac{\alpha \omega}{p} < (1 - \lambda^L) \theta \) implies that consumption to the lender in the pooling contract over \( \lambda^H \) is always larger or equal than in the pooling contract over \( \lambda^L \).

(c) otherwise, it is still the case that LC is slack and PC binds but \( c_1 = c_1^* > 0 \)

i. if \( c_{2h} = \frac{\alpha(\omega - c_1^*)}{p} > (1 - \lambda^L) \theta \) then (which is Case 2.b.iv in the pooling contract over \( \lambda^H \)):

\[ \begin{align*}
V^{bil, \lambda H} &= pu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - p) u \left( \omega - c_1^* \right) \\
V^{bil, \lambda L} &= pu \left( \omega + \lambda^L \theta \right) + (1 - p) u \left( \omega \right)
\end{align*} \]

so it is obvious that the pooling contract over \( \lambda^L \) is chosen. However this case can never arise, because we assumed that we are in the region of the parameter space where \( \omega < \frac{p(1 - \lambda^L) \theta}{\alpha} \).

ii. if \( c_{2h} = \frac{\alpha(\omega - c_1^{**})}{p} < (1 - \lambda^L) \theta \) then (which is Case 2.b.iii in the pooling contract over \( \lambda^H \)):

\[ \begin{align*}
V^{bil, \lambda H} &= p \left[ qu \left( \omega - c_1^{**} + \theta - \frac{\alpha(\omega - c_1^{**})}{p} \right) + (1 - q) u \left( \omega - c_1^{**} + \lambda^L \theta \right) \right] + \\
&\quad (1 - p) u \left( \omega - c_1^{**} \right) \\
V^{bil, \lambda L} &= pu \left( \omega + \lambda^L \theta \right) + (1 - p) u \left( \omega \right)
\end{align*} \]

Here there is a trade off: if and only if \( q \) is relatively small with respect to how risk averse lenders are, then the pooling contract over \( \lambda^L \) is preferred.

3. if \( \omega \geq \frac{p(1 - \lambda^L) \theta}{\alpha} \) then

(a) if \( \frac{\alpha - p}{1 - p} > \frac{u' \left( \frac{(1 - \lambda^L) \omega}{\alpha} \right)}{u' \left( \frac{\lambda^L \theta + (1 - \lambda^L) \omega}{\alpha} \right)} \) then both LC and PC bind in the pooling contract over \( \lambda^L \), and:

\[ V^{bil, \lambda L} = pu \left( \frac{1 - \lambda^L}{\alpha} \theta + \lambda^L \theta \right) + (1 - p) u \left( \frac{1 - \lambda^L}{\alpha} \theta \right) \]

i. if also:

\[ (\alpha - p) qu' \left( \frac{p(1 - \lambda^H) \theta}{\alpha} + \lambda^H \theta \right) > p(1 - q) u' \left( \frac{p(1 - \lambda^H) \theta}{\alpha} + \lambda^L \theta \right) \]
then both LC and PC bind in the pooling contract over $\lambda^H$ and:

$$V_{bil,\lambda H} = p \left[ qu \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^H \theta \right) + (1 - q) u \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^L \theta \right) \right] + (1 - p) u' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} \right)$$

$$V_{bil,\lambda L} = pu \left( \frac{(1 - \lambda^L) p \theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} \right)$$

Here there is a trade off: if and only if $q$ is relatively small with respect to how risk averse lenders are, then the pooling contract over $\lambda^L$ is preferred.

ii. if, however, $(\alpha - p) u' \left( \omega \left( 1 - \frac{2}{p} \right) + \theta \right) < (1 - p) u' (\omega)$ then LC is slack and PC binds in the pooling contract over $\lambda^H$, $c_{2h} = \frac{\alpha \omega}{p} > (1 - \lambda^L) \theta$, $c_1 = 0$ and:

$$V_{bil,\lambda H} = pu \left( \frac{(1 - \lambda^H) p \theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} \right)$$

However this case never arises because $(\alpha - p) u' \left( \omega \left( 1 - \frac{2}{p} \right) + \theta \right) < (1 - p) u' (\omega)$ contradicts $\frac{\alpha \omega}{1 - p} \geq \frac{u' \left( \frac{(1 - \lambda^L) p \theta}{\alpha} \right)}{u' \left( \lambda^L \theta + \frac{(1 - \lambda^L) p \theta}{\alpha} \right)}$.

iii. if $(\alpha - p) qu' \left( \omega \left( 1 - \frac{2}{p} \right) + \theta \right) < p (1 - q) u' (\omega + \lambda^L \theta) + (1 - p) u' (\omega)$ then still LC is slack and PC binds in the pooling contract over $\lambda^H$, but $c_{2h} = \frac{\alpha \omega}{p} < (1 - \lambda^L) \theta$, $c_1 = 0$ and:

$$V_{bil,\lambda H} = p \left[ qu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - q) u (\omega + \lambda^L \theta) \right] + (1 - p) u (\omega)$$

$$V_{bil,\lambda L} = pu \left( \frac{(1 - \lambda^L) p \theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} \right)$$

In this case there is a trade off if and only if $q$ is relatively small with respect to how risk averse lenders are, then the pooling con-

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19 Notice that $\omega < \frac{(1 - \lambda^L) p \theta}{\alpha}$ implies that $\omega + \theta - \frac{\alpha \omega}{p} > \frac{(1 - \lambda^L) p \theta}{\alpha} + \lambda^L \theta$. 

18
tract over $\lambda^L$ is preferred. However, this is not possible because we are in the region of the parameter space where $\omega \geq \frac{\alpha(\omega-c_1^*)}{p}$.

iv. otherwise, it is still the case that LC is slack and PC binds in the pooling contract over $\lambda^H$, but $c_1 = c_1^* > 0$, $c_{2h} = \frac{\alpha(\omega-c_1^*)}{p} > (1 - \lambda^L) \theta$ and:

\[
V_{bil,\lambda H} = pu\left(\omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p}\right) + (1 - p) u\left(\omega - c_1^*\right)
\]

\[
V_{bil,\lambda L} = pu\left(\frac{1 - \lambda^L}{\alpha} p^2 + \lambda^L \theta\right) + (1 - p) u\left(\frac{1 - \lambda^L}{\alpha} p^2\right)
\]

Here there is a trade off: if $c_1^*$ is relatively large then the pooling contract over $\lambda^L$ is preferred. However this can never happen: it can not be that $c_{2h} = \frac{\alpha(\omega-c_1^*)}{p} > (1 - \lambda^L) \theta$. Indeed, recall that $c_1^*$ and $c_{2h}$ are defined as

\[
\frac{\alpha - p}{1 - p} = \frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha(\omega-c_1^*)}{p})}
\]

\[
c_{2h} = \frac{\alpha(\omega-c_1^*)}{p}
\]

and that we have assumed that, when pooling over $\lambda^L$, the following condition holds:

\[
\frac{\alpha - p}{1 - p} > \frac{u'(\frac{(1-\lambda^L)p\theta}{\alpha})}{u'(\frac{\lambda^L\theta + (1-\lambda^L)p\theta}{\alpha})}
\]

Combining the two expressions we obtain

\[
\frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha(\omega-c_1^*)}{p})} = \frac{\alpha - p}{1 - p} > \frac{u'(\frac{(1-\lambda^L)p\theta}{\alpha})}{u'(\frac{\lambda^L\theta + (1-\lambda^L)p\theta}{\alpha})}
\]

(*)

Suppose then that $c_{2h} = \frac{\alpha(\omega-c_1^*)}{p} > (1 - \lambda^L) \theta$. Then,

\[
\omega - c_1^* > \frac{(1 - \lambda^L)p\theta}{\alpha} \implies u'(\omega - c_1^*) < u'(\frac{(1 - \lambda^L)p\theta}{\alpha})
\]

and

\[
\omega - c_1^* + \theta - \frac{\alpha(\omega-c_1^*)}{p} = \theta - \frac{\alpha - p}{p}(\omega - c_1^*)
\]
\[
< \theta - \frac{\alpha - p (1 - \lambda^L) \theta}{\alpha} = \lambda^L \theta + \frac{(1 - \lambda^L) \theta}{\alpha}
\]

\[= \Rightarrow u' \left( \omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p} \right) > u' \left( \lambda^L \theta + \frac{(1 - \lambda^L) \theta}{\alpha} \right) \]

Combining the two expressions we obtain

\[
\frac{u' (\omega - c_1^*)}{u' (\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p})} < \frac{\frac{u' \left( \frac{(1 - \lambda^L) \theta}{\alpha} \right)}{u' \left( \lambda^L \theta + \frac{(1 - \lambda^L) \theta}{\alpha} \right)}}
\]

that contradicts (*).

\[v. \text{ if } c_{2h} = \frac{\alpha (\omega - c_1^*)}{p} < (1 - \lambda^L) \theta \text{ then, it is still the case that LC is slack and PC binds in the pooling contract over } \lambda^H \text{ with } c_1 = c_1^* > 0 \text{, and:}
\]
\[
V^{bil, \lambda H} = p \left[ q u (\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p}) + (1 - q) u (\omega - c_1^* + \lambda^L \theta) \right] + (1 - p) u (\omega - c_1^*)
\]
\[
V^{bil, \lambda L} = p u \left( \frac{1 - \lambda^L}{\alpha} \frac{\theta}{\theta} + \lambda^L \theta \right) + (1 - p) u \left( \frac{(1 - \lambda^L) \theta}{\alpha} \right)
\]

Although this does not help in assessing which contract is preferred, notice that the expected repayment when pooling over \( \lambda^H \) is
\[
p\theta - (w - c_1^*) [\alpha q - 1] - (1 - q)(1 - \lambda^L) \theta
\]
while the expected repayment when pooling over \( \lambda^L \) is
\[
p\theta - \frac{(1 - \lambda^L) \theta}{\alpha} [\alpha - 1]
\]

Using the fact that \( \frac{\alpha (\omega - c_1^*)}{p} < (1 - \lambda^L) \theta \), we can show that the expected repayment when pooling over \( \lambda^H \) is larger than the expected repayment when pooling over \( \lambda^L \):
\[
p\theta - (w - c_1^*) [\alpha q - 1] - (1 - q)(1 - \lambda^L) \theta > p\theta - \frac{(1 - \lambda^L) \theta}{\alpha} [\alpha - 1]
\]

Therefore pooling over \( \lambda^H \) results in a lottery with a larger expected value.

Notice that, because the participation constraint binds in both contracts, \( c_{2h}^H < (1 - \lambda^L) \theta \) implies \( c_1^H > c_1^L \). Then, in state \( s = h \) we have that depending on risk aversion either the pooling contract over \( \lambda^H \) or over \( \lambda^L \) is preferred. In state \( s = l \) the pooling contract over \( \lambda^L \) is better given that \( \frac{\alpha (\omega - c_1^*)}{p} < (1 - \lambda^L) \theta \).
(b) if \( \frac{\alpha - p}{1-p} \leq \frac{u'(\omega)}{u'(\omega + \theta - \frac{\alpha \omega}{p})} \) then PC binds, LC is slack in the pooling contract over \( \lambda^L \), and \( c_1 = 0 \), so that:

\[
V_{bil,\lambda L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u(\omega)
\]

i. if also:

\[
(\alpha - p) qu' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^H \theta \right) > p(1 - q) u' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} \right)
\]

then both LC and PC bind in the pooling contract over \( \lambda^H \) and:

\[
V_{bil,\lambda H} = p \left[ qu \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^H \theta \right) + (1 - q) u \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^L \theta \right) \right] +
\]

\[
(1 - p) u \left( \frac{p (1 - \lambda^H) \theta}{\alpha} \right)
\]

\[
V_{bil,\lambda L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u(\omega)
\]

Here there is a trade off: if and only if \( q \) is relatively small and lenders are vert risk averse then the pooling contract over \( \lambda^L \) is preferred. This however is not possible because it requires \( \omega < \frac{p (1 - \lambda^H) \theta}{\alpha} \).

ii. if, however, \( (\alpha - p) u' \left( \omega \left( 1 - \frac{\alpha}{p} \right) + \theta \right) < (1 - p) u'(\omega) \) then LC is slack and PC binds in the pooling contract over \( \lambda^H \), \( c_{2h} = \frac{\alpha \omega}{p} > (1 - \lambda^L) \theta \), \( c_1 = 0 \) and

\[
V_{bil,\lambda H} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u(\omega)
\]

\[
V_{bil,\lambda L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u(\omega)
\]

So lenders are indifferent between the two contracts.

\[20\] Notice that this condition may still be satisfied at the same time as the sufficient condition for both constraints binding; in fact \( (\alpha - p) qu' \left( \omega \left( 1 - \frac{\alpha}{p} \right) + \theta \right) < p (1 - q) u' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^L \theta \right) \) and \( (\alpha - p) qu' \left( \omega \left( 1 - \frac{\alpha}{p} \right) + \theta \right) < (\alpha - p) qu' \left( \frac{p (1 - \lambda^H) \theta}{\alpha} + \lambda^H \theta \right) \). But, when both conditions are satisfied, the solution is the one with LC slack.
This is because \( c_1 = 0 \) in both contracts and the limited commitment constraint in slack in the pooling contract over \( \lambda^H \) to the point that it is also slack for a \( \lambda^L \) type of borrower. Thus the allocation in every state of the world is the same under either contract.

iii. if \( (\alpha - p) qu' \left( \omega \left(1 - \frac{\alpha}{p} \right) + \theta \right) < p (1 - q) u' \left( \omega + \lambda^L \theta \right) + (1 - p) u' \left( \omega \right) \) then still LC is slack and PC binds in the pooling contract over \( \lambda^H \), but \( c_2h = \frac{\alpha \omega}{p} < (1 - \lambda^L) \theta, c_1 = 0 \) and:

\[
V^{bil,H} = p \left[ qu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - q) u \left( \omega + \lambda^L \theta \right) \right] + (1 - p) u \left( \omega \right)
\]

\[
V^{bil,L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u \left( \omega \right)
\]

In this case the pooling contract over \( \lambda^L \) is preferred. This however is not possible because we are in the region \( \omega \geq \frac{p(1 - \lambda^L) \theta}{\alpha} \).

iv. otherwise, it is still the case that LC is slack and PC binds in the pooling contract over \( \lambda^H \), but \( c_1 = c_1^* > 0, c_2h = \frac{\alpha(\omega - c_1^*)}{p} > (1 - \lambda^L) \theta \) and:

\[
V^{bil,H} = pu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - p) u \left( \omega - c_1^* \right)
\]

\[
V^{bil,L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u \left( \omega \right)
\]

Here there is no trade off: the pooling contract over \( \lambda^L \) is preferred. The pooling over \( \lambda^H \) contract is a mean-preserving-spread of the pooling over \( \lambda^L \) contract.

v. if \( c_2h = \frac{\alpha(\omega - c_1^*)}{p} < (1 - \lambda^L) \theta \) then, it is still the case that LC is slack and PC binds in the pooling contract over \( \lambda^H \) with \( c_1 = c_1^* > 0 \), and:

\[
V^{bil,H} = p \left[ qu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - q) u \left( \omega - c_1^* + \lambda^L \theta \right) \right] + (1 - p) u \left( \omega - c_1^* \right)
\]

\[
V^{bil,L} = pu \left( \omega + \theta - \frac{\alpha \omega}{p} \right) + (1 - p) u \left( \omega \right)
\]

However this can never happen: recall that we assumed

\[
\frac{\alpha - p}{1 - p} \leq \frac{u'(\omega)}{u' \left( \theta - \frac{\alpha - p}{p} \omega \right)} \tag{**}
\]

Using the fact that \( \frac{\alpha(\omega - c_1^*)}{p} < (1 - \lambda^L) \theta \), it follows that

\[
p(1 - q)u'(\omega - c_1^* + \lambda^L \theta) = p(1 - q)u' (\omega - c_1^* + \theta - (1 - \lambda^L) \theta)
\]
> p(1 - q)u' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right)

that replace in the optimality condition for \( c_1^{**} \) gives

\[
(\alpha - p)qu' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right) = p (1 - q) u' (\omega - c_1^{**} + \lambda^L \theta) + (1 - p) u' (\omega - c_1^{**})
\]

\[
> p(1 - q)u' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right) + (1 - p) u' (\omega - c_1^{**})
\]

Rearranging:

\[
u' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right) \left[ (\alpha - p)q - p(1 - q) \right] > (1 - p) u' (\omega - c_1^{**})
\]

and therefore

\[
(\alpha - p)u' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right) > (1 - p) u' (\omega - c_1^{**}) \quad (***)
\]

Combining (**) with (***) we obtain

\[
\frac{u' (\omega)}{u' \left( \theta - \frac{\alpha - p}{p} \omega \right)} \geq \frac{\alpha - p}{1 - p} > \frac{u' (\omega - c_1^{**})}{u' \left( \theta - \frac{\alpha - p}{p} (\omega - c_1^{**}) \right)}
\]

which, by concavity of \( u(\cdot) \) requires \( c_1^{**} < 0 \), which is a contradiction.

In this case there is a trade off: in state \( s = l \) the pooling contract over \( \lambda^L \) is preferred because \( c_2^{HL} < (1 - \lambda^L) \theta \) implies \( c_1^{HL} < c_1^{HL} \) since the participation constraint is binding in both contracts. in state \( s = h \), however, a sufficient condition for the pooling contract over \( \lambda^L \) to be preferred is

\[
-c_1^{**} + q[\theta(1 - \lambda^L) - \frac{\alpha}{p}(\omega - c_1^{**})] < \theta(1 - \lambda^L) - \frac{\alpha}{p} \omega
\]

In fact, if this is true, then we have:

\[
qu \left( \omega - c_1^{**} + \theta - \frac{\alpha(\omega - c_1^{**})}{p} \right) + (1 - q) u (\omega - c_1^{**} + \lambda^L \theta) < \]

\[
u \left[ q \left( \omega - c_1^{**} + \theta - \frac{\alpha(\omega - c_1^{**})}{p} \right) + (1 - q) (\omega - c_1^{**} + \lambda^L \theta) \right] < \]

\[
u \left( \omega + \theta - \frac{\alpha \omega}{p} \right)
\]

Otherwise there is a trade-off between the two contracts and other conditions will resolve it.
Otherwise still PC binds, LC is slack in the pooling contract over $\lambda^L$, but $c_1 > 0$, it solves $\frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p})}$, and:

$$V_{bil,\lambda L} = pu \left( \omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p} \right) + (1 - p) u(\omega - c_1^*)$$

i. if also:

$$(\alpha - p) qu' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^H \theta \right) > p \left(1 - q\right) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^L \theta \right)$$

$$+ (1 - p) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} \right)$$

then both LC and PC bind in the pooling contract over $\lambda^H$ and:

$$V_{bil,\lambda H} = p qu \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^H \theta \right) + (1 - q) u \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^L \theta \right)$$

$$+ (1 - p) u \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} \right)$$

$$V_{bil,\lambda L} = pu \left( \omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p} \right) + (1 - p) u(\omega - c_1^*)$$

However this can never happen: for the PC to bind and LC to be slack in the pooling contract over $\lambda^L$ it must be that

$$\frac{\alpha (\omega - c_1^*)}{p} \geq (1 - \lambda^L) \theta > (1 - \lambda^H) \theta$$

$$\implies \omega - c_1^* > \frac{(1 - \lambda^H) p \theta}{\alpha}$$

Using this and concavity of $u$ we obtain

$$p \left(1 - q\right) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^L \theta \right) > p \left(1 - q\right) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^H \theta \right)$$

that replaced in the optimality condition for the pooling contract of $\lambda^H$ gives

$$(\alpha - p) qu' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^H \theta \right)$$

$$> p \left(1 - q\right) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} + \lambda^L \theta \right) + (1 - p) u' \left( \frac{p \left(1 - \lambda^H\right) \theta}{\alpha} \right)$$

24
\[ p(1 - q)u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right) + (1 - p)u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} \right) \]

and therefore, after rearranging,

\[(\alpha - p)u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right) > [\alpha - p - p(1 - q)]u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right) > (1 - p)u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} \right) \]

or

\[ \frac{\alpha - p}{1 - p} > \frac{u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} \right)}{u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right)} \quad (*) \]

Recall now the optimality condition of pooling over \( \lambda \) we obtain

\[ \frac{u' (\omega - c_1^*)}{u' (\omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p})} = \frac{\alpha - p}{1 - p} \quad (**) \]

For the LC constraint to be slack, it has to be that

\[ \frac{\alpha(\omega - c_1^*)}{p} \geq (1 - \lambda^L)\theta > (1 - \lambda^H)\theta \]

\[ \Rightarrow \omega - c_1^* > \frac{(1 - \lambda^H)p\theta}{\alpha} \]

\[ \Rightarrow u' (\omega - c_1^*) < u' \left( \frac{(1 - \lambda^H)p\theta}{\alpha} \right) \]

\[ \Rightarrow u' \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) = u' \left( \theta - \frac{\alpha - p}{p} (w - c_1^*) \right) > u' \left( \theta - \frac{\alpha - p (1 - \lambda^H)p\theta}{\alpha(\omega - c_1^*)} \right) = u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right) \]

that replaced in (**) gives

\[ \frac{\alpha - p}{1 - p} = \frac{u' (\omega - c_1^*)}{u' (\omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p})} < \frac{u' \left( \frac{(1 - \lambda^H)p\theta}{\alpha} \right)}{u' \left( \frac{p(1 - \lambda^H)\theta}{\alpha} + \lambda^H\theta \right)} \]

that contradicts (*).

Here there is a trade off: if and only if \( q \) is relatively small with
respect to how risk averse lenders are, then the pooling contract over \( \lambda^L \) is preferred.\(^{21}\)

ii. if, however, \((\alpha - p) u'(\omega (1 - \frac{\omega}{p}) + \theta) < (1 - p) u'(\omega)\) then LC is slack and PC binds in the pooling contract over \( \lambda^H \), \( c_{2h} = \frac{\alpha \omega}{p} > (1 - \lambda^L) \theta \), \( c_1 = 0 \) and

\[
V^{bl,\lambda H} = pu(\omega + \theta - \frac{\alpha \omega}{p}) + (1 - p) u(\omega) \\
V^{bl,\lambda L} = pu(\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p}) + (1 - p) u(\omega - c_1^*)
\]

This case, however, can never arise because:

\[
\frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha}{p}(\omega - c_1^*))} > \frac{u'(\omega)}{u'(\omega - \frac{\alpha}{p} + \theta)}
\]

Thus it is impossible to have at the same time:

\[
\frac{u'(\omega)}{u'(\omega - \frac{\alpha}{p} + \theta)} > \frac{\alpha - p}{1 - p}
\]

and

\[
\frac{u'(\omega - c_1^*)}{u'(\omega - c_1^* + \theta - \frac{\alpha}{p}(\omega - c_1^*))} = \frac{\alpha - p}{1 - p}
\]

iii. if \((\alpha - p) qu'(\omega (1 - \frac{\omega}{p}) + \theta) < q(1 - q) u'(\omega + \lambda^L \theta) + (1 - p) u'(\omega)\) then still LC is slack and PC binds in the pooling contract over \( \lambda^H \), but \( c_{2h} = \frac{\alpha \omega}{p} < (1 - \lambda^L) \theta \), \( c_1 = 0 \) and:

\[
V^{bl,\lambda H} = pq u(\omega + \theta - \frac{\alpha \omega}{p}) + (1 - q) u(\omega + \lambda^L \theta) + (1 - p) u(\omega) \\
V^{bl,\lambda L} = pu(\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p}) + (1 - p) u(\omega - c_1^*)
\]

This case, however, can never arise because we assumed that \( \omega > (1 - \lambda^L) \frac{p q}{\alpha} \), contradicting \( c_{2h} = \frac{\alpha \omega}{p} < (1 - \lambda^L) \theta \).

iv. otherwise, it is still the case that LC is slack and PC binds in the pooling contract over \( \lambda^H \), but \( c_1 = c_1^* > 0 \), \( c_{2h} = \frac{\alpha \omega - c_1^*}{p} > (1 - \lambda^L) \theta \) and:

\[
V^{bl,\lambda H} = pu(\omega - c_1^* + \theta - \frac{\alpha (\omega - c_1^*)}{p}) + (1 - p) u(\omega - c_1^*)
\]

\(^{21}\) The trade-off arises because \( c_1^{L_1} < c_1^{H_1} \) given that the participation constraint binds in both contracts but \( c_{2h}^{L_1} > (1 - \lambda^L) \theta > c_{2h}^{H_1} = (1 - \lambda^H) \theta \), also implying that \( c_{2h}^{L_1} > c_{2h}^{H_1} + (\lambda^H - \lambda^L) \theta \). Therefore the pooling contract over \( \lambda^L \) is preferred in state \( s = l \) but the pooling contract over \( \lambda^H \) is preferred in state \( s = h \).
\[ V^{bil,\lambda L} = pu \left( \omega - c_1^* + \theta - \frac{\alpha(\omega - c_1^*)}{p} \right) + (1 - p) u(\omega - c_1^*) \]

So the two contracts are payoff equivalent to lenders.

v. if \( c_{2h} = \frac{\alpha \omega - c_1^{**}}{p} < (1 - \lambda^L) \theta \) then, it is still the case that LC is slack and PC binds in the pooling contract over \( \lambda^H \) with \( c_1 = c_1^* > 0 \), and:

\[ V^{bil,\lambda H} = p \left[ qu \left( \omega - c_1^{**} + \theta - \frac{\alpha \omega - c_1^{**}}{p} \right) + (1 - q) u(\omega - c_1^{**} + \lambda^L \theta) \right] + (1 - p) u(\omega - c_1^*) \]

In this case there is a trade off: since \( c_1^{**} < c_1^* \) then if and only if \( p \) is large and lenders are very risk averse then the pooling contract over \( \lambda^L \) is preferred. \( c_1^{**} < c_1^* \) because the participation constraint binds in both contracts, but \( c_2^{Hh} < c_2^{Lh} \) by assumption.

4 CCP decision problem

Suppose that the CCP maximizes lenders’ welfare\(^{22}\) using the resources available in both time periods: suppose that both agents have to transfer their beginning of period endowments to the CCP and then they receive consumption payments, based on the contract they submit to the CCP for clearing.\(^{23}\)

We first characterize the contract where the CCP offer lenders (and borrowers) a pooling contract over \( \lambda^L \) and in the following sections we analyze the case of pooling contract over \( \lambda^H \) and separating contract. However, the economies we are mostly interested in are the ones where in equilibrium the CCP and lenders choose the pooling contract over \( \lambda^L \) because we want to compare its outcome with the outcome of an equilibrium with bilateral clearing and information acquisition. An economy where the equilibrium with CCP clearing is the pooling

\(^{22}\)Notice that \( c_1^* \) and \( c_1^{**} \) solve, respectively: \( u' \left( \omega - c_1 + \theta - \frac{\alpha \omega - c_1}{p} \right) = \frac{(1-p)}{(1-q)} u'(\omega - c_1) \) and \( u'(\omega - c_1 + \theta - \frac{\alpha \omega - c_1}{p}) = \frac{1}{(1-p)(1-q)} \left[ (1-p) u'(\omega - c_1) + p(1-q) u'(\omega - c_1 + \lambda^L \theta) \right] \). Because the left hand side of both equations is decreasing in \( c_1 \) but the right hand side is larger in the case of \( c_1^{**} \) which implies \( c_1^{**} < c_1^* \).

\(^{23}\)One could assume that this is a repeated game and the role of borrower and lender switches randomly (or not) so that even borrowers are happy to join the CCP since eventually they take advantage of it.

\(^{24}\)We still need to prove that this is equivalent to having agents submitting a contract to the CCP and the CCP charging default fund contributions and margins accordingly. Agents would take into account such contributions and margin allocation rules when they agree on a contract, and then play their best response to such rules by choosing the terms of trade. Then the terms of trade are transferred to the CCP and the contributions and margining rules are applied. The conjecture is that such contributions and margining rules would implement the allocation that solves our CCP decision problem.
over $\lambda^L$ and the equilibrium with bilateral clearing is the one with information acquisition, is an economy where CCP clearing induces loss of information.

With central clearing, each lender gets randomly matched with one borrower, that can be of type $\{\lambda_L, \lambda_H\}$. The lender decides whether to verify counterparty type, $e = 1$ or not $e = 0$. After verification, the lender reports the type of the counterparty, $H, L, \emptyset$, where $\emptyset$ means he did not observe. The only two things that are contractible for the CCP are the message $m_L$ and the second period idiosyncratic state of a borrower $\{h, l\}$. After the message has been sent, the borrower can accept or he can reject the transaction. If the borrower rejects, both lender and borrower consume autarky payoff. If accepted, the CCP becomes counterparty of both lenders and borrowers. If the transaction is accepted, borrowers transfer the consumption good to the CCP, and lenders transfer the capital good to borrowers.

If $m_L \neq \emptyset$, the CCP gives consumption $C^{m_L}_1$ to borrowers, where $m_L$ is the message sent by the lender in the first period. In the second period, the idiosyncratic state $\{h, l\}$ realizes. The CCP asks borrowers to transfer $0$ if the state is low, and to transfer $\theta$ if the state is high. In high state, the borrowers can refuse to pay the CCP the requested amount, getting away with $(1 - \lambda_i) \theta$. Out of its total revenues, the CCP then transfers consumption $X^{m_L}_2$ to lenders, and $C^{m_L}_{2,H}$ to borrowers if the state is high, and $C^{m_L}_{2,L}$ if the state is low.

If $m_L = \emptyset$, the CCP offers a contract $X^\emptyset_2$ to lenders and a menu of contracts $(C^{hi}_{1,2}, C^{hi}_{2,h}, C^{hi}_{2,l})$, $(C^{lo}_{1,2}, C^{lo}_{2,h}, C^{lo}_{2,l})$. This is shown in Figure 2.

Lemma 5 The only incentive compatible contract, has the lender revealing that he did not learn the counterparty type: $m_L = \emptyset$. As a consequence $X^H_2 = X^L_2$.

Proof. By contradiction. Suppose there is an incentive compatible contract with the lender exerting monitoring effort, and then revealing the information acquired. Then, after exerting effort,

1. When the lender learns $\lambda_i = \lambda_L$, he must truthfully report it: if $\alpha C^H_1 + p \max\{C^H_{2,h}, (1 - \lambda_L) \theta\} + (1 - p) C^H_{2,l} \geq \alpha \omega$, that is to say the contract offered to a $\lambda_L$ type satisfies the borrower’s participation constraint, then:

   $X^L_2 \geq X^H_2$

Since

$\alpha C^H_1 + p \max\{C^H_{2,h}, (1 - \lambda_L) \theta\} + (1 - p) C^H_{2,l} \geq \alpha C^H_1 + p C^H_{2,h} + (1 - p) C^H_{2,l} \geq \alpha \omega$

then an incentive compatible contract must satisfy: $X^L_2 \geq X^H_2$.

2. Similarly, when the lender learns $\lambda_i = \lambda_H$, he must truthfully report it: if $\alpha C^L_1 + p \max\{C^L_{2,h}, (1 - \lambda_H) \theta\} + (1 - p) C^L_{2,l} \geq \alpha \omega$, that is to say the contract offered to a $\lambda_L$ type satisfies the borrower’s participation constraint, then:

   $X^H_2 \geq X^L_2$
Since
\[ \alpha C^L_i + p \max \{ C^L_{2,h}, (1 - \lambda_H)\theta \} + (1 - p) C^L_{2,l} \geq \alpha C^L_i + p C^L_{2,h} + (1 - p) C^L_{2,l} \geq \alpha \omega \]
then an incentive compatible contract must satisfy: \( X^H_2 \geq X^L_2 \).

Combining the two, \( X^L_2 = X^H_2 \). Then, in order to have the lender verify the counterparty type, rather than just reporting \( m_L(\emptyset) = \lambda_H \), it must be that his payoff is weakly larger, assuming that the participation constraint of the \( \lambda_H \) type is satisfied. That is to say, if \( \alpha C^H_i + p \max \{ C^H_{2,h}, (1 - \lambda_H)\theta \} + (1 - p) C^H_{2,l} \geq \alpha \omega \), the lender acquires information about the borrower’s type if:
\[ qu(X^H) + (1 - q) u(X^L_2) - \gamma = u(X^H_2) - \gamma \geq u(X^H) \]
which, however, is never satisfied. Therefore, because the participation constraint of the \( \lambda_H \) type is satisfied, (i.e. \( \alpha C^H_i + p \max \{ C^H_{2,h}, (1 - \lambda_H)\theta \} + (1 - p) C^H_{2,l} \geq \alpha \omega \)), then the lender never acquires information about the borrower’s type.

link this to the discussion about reporting nothing in the previous page. Hence, the only contract we need to characterize is the one in which the CCP offers a menu of contracts \( (C^H_i, C^L_{2,h}, C^L_{2,l}) \), \( (C^H_{1,i}, C^H_{2,h}, C^L_{2,l}) \) in which borrowers have incentives to self-select.

The CCP solves:\(^{25}\)

\[ \begin{align*}
(P1) \quad \max \quad & qu \left( X^H_i \right) + (1 - q) \left( X^L_2 \right) \\
\text{s.t.} \quad & \alpha C^H_i + p \max \{ C^H_{2,h}, (1 - \lambda_H)\theta \} + (1 - p) C^H_{2,l} \geq \alpha \omega, \quad \forall i \\
& q C^H_i + (1 - q) C^L_i \leq \omega 
\end{align*} \]

\(^{25}\)FOR US. The constraint with the \( \beta^i \) multiplier, as we show in the lemma in the next page, puts a lower bound on the consumption \( C^H_{2,h} \) and \( C^L_{2,h} \) of both types of borrowers: the limited commitment problem of both borrowers are flatten to \( 1 - \lambda^L \). In turn, this produces an interaction between private information and limited commitment that is vital for the existence of a trade-off between central and bilateral clearing. Indeed, if instead of \( \max \{ C^e_{2,h}, (1 - \lambda^L)\theta \} \) we just had \( C^e_{2,h} \), such an interaction would disappear: the constraint with \( \beta^i \) would always be irrelevant in the CCP problem, since the CCP could elicit information from borrowers at no cost. A similar argument would apply to the bilateral problem: a lender can elicit information at no cost, and he would never choose to pay the cost \( \gamma \). Because the asymmetric information problem would disappear, the trade-off between central and bilateral clearing would disappear as well: CCP clearing would always dominate bilateral clearing. Then, writing the constraint \( \beta^i \) with \( \max \{ C^e_{2,h}, (1 - \lambda^L)\theta \} \) on the RHS, is vital for the existence of a trade-off between bilateral and central clearing, since this makes the information asymmetry relevant to the problem. How can we justify this constraint? One interpretation is that \( \lambda^L \) is a short-cut for a moral-hazard problem a’ la Holmstrom Tirole: borrowers have an opportunity cost of being diligent, where being diligent reduces the probability of failure (but we still have full support). Nevertheless, if the borrower has been diligent or not can’t be fully detected ex-post. In this sense, we can reinterpret the heterogeneity in \( \lambda^L \) as heterogeneity in the opportunity cost of being diligent; this parameter can be learned by market participants, but it is more difficult to be learned by the CCP.
\[ q \left( X_2^H + pC_{2,h} + (1-p)C_{2,l}^H \right) + (1-q) \left( X_2^L + pC_{2,h} + (1-p)C_{2,l}^L \right) \leq \omega - qC_1^H - (1-q)C_1^L + p\theta \]
\[ C_{2,h} \geq (1-\lambda^i)\theta, \quad \forall i \] (14)
\[ \alpha C_{1,i} + pC_{1,h} + (1-p)C_{1,l} \geq 0 \]
\[ \alpha C_{1,i}^T + p \max (C_{2,h}^- (1-\lambda^i)\theta) + (1-p)C_{2,l}^- \quad \forall i \] (16)

Then let \( \mu_i \) denote the Lagrange multiplier on participation constraints \((12)\), \( \rho_1 \) and \( \rho_2 \) the multipliers on the resource constraints in period 1, \((13)\), and in period 2, \((14)\), \( \eta_i \) the multiplier on the limited commitment constraints \((15)\) and \( \beta_i \) the multiplier on the incentive constraints \((16)\).

Notice that the CCP could make payments to lenders conditional on the realization of the borrower’s state: \( X_{2,h}^H, X_{2,l}^H \) and \( X_{2,h}^L, X_{2,l}^L \). However, because the CCP pools resources across all borrowers then it can offer insurance against that shock. With a risk averse lender this will always yield higher utility than making different payments contingent on the realization of the shock. Thus \( X_{2,h}^H = X_{2,l}^H \) and \( X_{2,h}^L = X_{2,l}^L \).

Then, we can substitute \( X_2^H = X_2^L = X_2 \) in the objective function of \((P1)\). Also, the feasibility constraint in period 2 \((14)\) must bind: if not, then the CCP can increase \( X_2 \) and attain a higher value of the objective without violating any of the constraints. Then, we use in the objective function of \((P1)\):

\[ X_2 = \omega - q \left( C_1^H + pC_{1,h} + (1-p)C_{1,l}^H \right) - (1-q) \left( C_1^L + pC_{1,h} + (1-p)C_{1,l}^L \right) \]

Finally, since maximizing \( u(X_2) \) is equivalent to maximizing \( X_2 \), we can consider the CCP as de facto being risk-neutral. Finally, for \( i = L, H \), constraints \((16)\) and \((15)\) can be rewritten in a more compact form, as the following lemma shows.

**Lemma 6** Constraints \((16)\) and \((15)\) hold if and only if

\[ \min \{ C_{2,h}^H, C_{2,h}^L \} \geq (1-\lambda^L)\theta \quad \text{(17a)} \]

\[ \alpha C_{1,h}^H + pC_{1,h}^L + (1-p)C_{1,l}^H = \alpha C_{1,l}^H + pC_{1,h}^L + (1-p)C_{1,l}^L \quad \text{(17b)} \]

**Proof.** Suppose first constraints \((16)\) and \((15)\) hold. Consider \((17a)\) first. Because \( \lambda^L < \lambda^H \), from \((16)\) for \( i = L \) we have \( C_{2,h}^L \geq (1-\lambda^L)\theta > (1-\lambda^H)\theta \). We are now left to prove that \( C_{2,h}^H \geq (1-\lambda^L)\theta \). Notice that constraint \((16)\) for \( i = H \) becomes

\[ \alpha C_{1,h}^H + pC_{1,h}^L + (1-p)C_{1,l}^H \geq \alpha C_{1,l}^H + pC_{1,h}^L + (1-p)C_{1,l}^L \quad \text{(18)} \]

because \( C_{2,h}^L \geq (1-\lambda^H)\theta \). Combining this with \((16)\) for \( i = L \) we obtain

\[ \alpha C_{1,h}^H + pC_{1,h}^L + (1-p)C_{1,l}^H \geq \alpha C_{1,h}^L + pC_{1,h}^L + (1-p)C_{1,l}^L \]

\[ \geq \alpha C_{1,h}^H + p \max \{ C_{2,h}^H, (1-\lambda^L)\theta \} + (1-p)C_{1,l}^H \]

\[ \iff C_{2,h}^H \geq \max \{ C_{2,h}^H, (1-\lambda^L)\theta \} \]

\[ \iff C_{2,h}^H \geq (1-\lambda^L)\theta \]
hence we proved (17a) holds. For (17b) notice that constraint (16) for \( i = L \) becomes
\[
\alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L \geq \alpha C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H
\]  
(19)
Combining (19) with (18) we obtain (17b).
Suppose now (17a) and (17b) hold. (17a) immediately implies that (15) for \( i = L, H \) are satisfied because \( C_{2,h}^H \geq (1 - \lambda^H) \theta > (1 - \lambda^H) \theta \) for \( i = L, H \). Then, because \( C_{2,h}^L = \max\{C_{2,h}^L, (1 - \lambda^L) \theta\} \) we can rewrite (17b) as:
\[
\alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L = \alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L
\]
\[
= \alpha C_1^L + p \max\{C_{2,h}^L, (1 - \lambda^L) \theta\} + (1 - p)C_{2,l}^L
\]
implying that constraint (16) for \( i = H \) is satisfied. Similarly, because \( C_{2,h}^H = \max\{C_{2,h}^H, (1 - \lambda^L) \theta\} \) then we can rewrite (17b) as:
\[
\alpha C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H = \alpha C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H
\]
\[
= \alpha C_1^H + p \max\{C_{2,h}^H, (1 - \lambda^L) \theta\} + (1 - p)C_{2,l}^H
\]
constraint (16) for \( i = L \) is satisfied.

From (17b) we can additionally ignore one participation constraint, say \((\mu)^H\), and rewrite the problem as:

\[
(P2) \quad \max_{(C_1, C_{2,h}, C_{2,l})} \omega + \rho \theta - \left[q(C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H) + (1 - q)(C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L)\right]
\]
\[\text{s.t.} \quad \alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L \geq \alpha \omega \]
\[
q C_1^H + (1 - q) C_1^L \leq \omega
\]
\[
\min\{C_{2,h}^H, C_{2,l}^H\} \geq (1 - \lambda^L) \theta
\]
\[
\alpha C_1^H + pC_{2,h}^H + (1 - p)C_{2,l}^H = \alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L
\]
\[
C_{2,h}^L, C_{2,l}^L \geq 0
\]

Let \( \mu \) denote the Lagrange multiplier on the participation constraint (20), \( \rho \) the multiplier on the resource constraint in period 1, (21), \( \eta \) the multiplier on the limited commitment constraint (22) and \( \beta \) the multiplier on the incentive constraint (23).

**Lemma 7** If \( \min\{C_{2,h}^H, C_{2,l}^H\} > (1 - \lambda^L) \theta \), the resource constraint in period 1 must be slack: \( q C_1^H + (1 - q) C_1^L < \omega \).

**Proof.** Suppose not. Then \( q C_1^H + (1 - q) C_1^L = \omega \) because \( \min\{C_{2,h}^H, C_{2,l}^H\} > (1 - \lambda^L) \theta \). Then either \( C_1^H > \omega \), \( C_1^L > \omega \) or both. Suppose it is \( C_1^H > \omega \), since the case \( C_1^L > \omega \) is analogous. Then \( \alpha C_1^L + pC_{2,h}^L + (1 - p)C_{2,l}^L > \alpha \omega \) and the participation constraint (20) must be slack. Then the CCP can decrease \( C_{2,h}^H \) and \( C_{2,l}^H \) by \( \epsilon > 0 \) arbitrarily small, so that all constraints are still satisfied and expected revenues of the CCP increase. ■
Lemma 8 If \( \min\{C_{2,h}^H, C_{2,L}^L\} > (1 - \lambda^L)\theta \), the resource constraint must bind: 
\[ qC_i^H + (1-q)C_i^L = \omega. \]

**Proof.** Suppose not. Then \( qC_i^H + (1-q)C_i^L < \omega \) because \( \min\{C_{2,h}^H, C_{2,L}^L\} > (1 - \lambda^L)\theta \). Consider the allocation \( C_i^L = C_i^L + \epsilon \) and \( C_i^L = C_i^L + \frac{\epsilon}{\alpha} \). This allocation satisfies all the constraints and yields higher value of the objective in \((P2)\).

Combining the two results above we then have the following proposition.

**Proposition 9** Constraint \( \square \) always binds: \( \min\{C_{2,h}^H, C_{2,L}^L\} = (1 - \lambda^L)\theta. \)

Lemma 10 Consumption in the second period low state equals zero for both type of borrowers: \( C_{2,i}^H = C_{2,i}^L = 0. \)

**Proof.** By contradiction. Consider first the case \( C_{2,i}^H > 0 \) and \( C_{2,i}^L > 0 \).

**STEP 1.**
\( C_{2,i}^H > 0 \) and \( C_{2,i}^L > 0 \) is not a solution to \((P2)\).

If both \( C_{2,i}^H > 0 \) and \( C_{2,i}^L > 0 \), then the participation constraint must bind, 
\[ \alpha C_i^L + pC_{2,h}^L + (1-p)C_{2,i}^L = \alpha \omega: \] otherwise we could just decrease \( C_{2,i}^H \) and \( C_{2,i}^L \) by the same amount increasing expected revenues. But then, given that the participation constraint binds, it must be that \( C_i^H < \omega \) and \( C_i^L < \omega \), hence 
\[ qC_i^H + (1-q)C_i^L < \omega \] and the feasibility constraint is slack. Then consider the allocation \( C_i^L = C_i^L + \epsilon \frac{(1-p)}{\alpha} \) and \( C_{2,i}^L = C_{2,i}^L - \epsilon \). This allocation is in the constraint set of \((P2)\) and yields higher value of the objective because it uses fewer resources given that \( \frac{(1-p)}{\alpha} < 1 \). Then \( C_{2,i}^L > C_{2,i}^L = 0 \) either for \( i = H \) or \( i = L. \)

**STEP 2.**
If \( C_{2,i}^L > C_{2,i}^L = 0 \) then the participation constraint is slack.

Suppose not: then the participation constraint binds, and \( C_i^H < \omega \), \( C_i^L < \omega \) because the limited commitment constraint \( \square \) implies \( C_{2,h}^H > 0 \) and \( C_{2,h}^L > 0 \). Then 
\[ qC_i^H + (1-q)C_i^L < \omega. \] Consider then the alternative allocation \( C_{2,i}^L = C_{2,i}^L - \epsilon \) and \( C_i^L = C_i^L + \epsilon \frac{(1-p)}{\alpha} \). This allocation is still in the constraint set of \((P2)\) for \( \epsilon > 0 \) arbitrarily small. It also yields higher value of the objective because \( \frac{(1-p)}{\alpha} < 1 \). Then the participation constraint must be slack.

**STEP 3.**
If \( C_{2,i}^L > C_{2,i}^L = 0 \) then \( C_i^H = C_i^L = 0. \)

Suppose not. Then \( C_i^L = 0 \) and \( C_i^L > 0 \). Because \( C_{2,i}^L > C_{2,i}^L = 0 \) by assumption, then STEP 2 implies that the participation constraint is slack. Then consider the alternative allocation \( C_{2,i}^L = C_{2,i}^L - \epsilon \) and \( C_i^L = C_i^L - \epsilon \frac{(1-p)}{\alpha} \) so that \( \square \) is still satisfied. Because the participation constraint \( \square \) was slack then for \( \epsilon > 0 \) arbitrarily small, then the allocation \( C_{2,i}^L, C_i^L \) satisfies the participation constraint \( \square \). Because the allocation \( C_{2,i}^L, C_i^L \) uses fewer resources then it must yield higher value of the objective to \((P2)\).

Suppose instead \( C_i^L = 0 \) and \( C_i^L > 0 \) when \( C_{2,i}^L > C_{2,i}^L = 0 \). Then STEP 2 implies that the participation constraint is slack, and constraints \( \square \) and \( \square \)
consider the alternative allocation $C_{2,h} = (1 - \lambda^L)\theta$. Then the CCP can offer the alternative allocation $C_i' = C_i - \frac{p_i}{\alpha}$ and $C_{2,h} = C_{2,h} - \varepsilon$ so that constraint (23) is still satisfied. For $\varepsilon > 0$ arbitrarily small than the participation constraint (20) and the limited commitment constraint (22) are still satisfied. Because the allocation $C_1', C_{2,h}'$ uses fewer resources then it yields higher value of the objective. Therefore, if the participation constraint (20) is slack then

$$C_i' = C_i = 0.$$  

**STEP 4.**

If $C_i' = C_i = 0$ then $C_{2,i} = C_{2,i} = 0$

Suppose not. Proposition 9 implies that $C_{2,i} > C_{2,h} = (1 - \lambda^L)\theta$. Then consider the alternative allocation $C_{2,h} = C_{2,h} - \varepsilon$ and $C_i' = C_i - C_i + \varepsilon \frac{p_i}{\alpha}$. This allocation is still in the constraint set of (P2) but uses fewer resources because $\frac{p_i}{\alpha} < 1$. Thus it yields a higher value of the objective. Therefore, $C_{2,i} = 0$ for all $i = L, H$.  

**Lemma 11** The solution of the CCP problem satisfies $C_i' = C_i = (1 - \lambda^L)\theta$.

**Proof.** By contradiction. Suppose that $C_i' = C_i > C_{2,h} = (1 - \lambda^L)\theta$.

By proposition 9, $\min \{ C_i' = C_i, C_{2,h} \} = (1 - \lambda^L)\theta$. Then, constraint (23) implies that $C_i' > C_i \geq 0$.

Suppose first that the participation constraint (20) is slack: then consider the alternative allocation $C_{2,h} = C_{2,h} - \varepsilon$ and $C_i' = C_i - \varepsilon \frac{p_i}{\alpha}$ so that (23) is still satisfied. The resource constraint (21) is also still satisfied because $C_i' < C_i$, and for $\varepsilon > 0$ arbitrarily small then the participation constraint (20) is satisfied because it was slack. Finally, the limited commitment constraint (22) is satisfied by assumption. Then the allocation $C_i', C_{2,h}$ is in the constraint set of problem (P2) but uses fewer resources, thus it achieves a higher value of the objective. Therefore, if the participation constraint (20) is slack then $C_i' > C_i \geq 0$ is a solution to (P2).

Suppose now that the participation constraint (20) is binding. Because $\min \{ C_i' = C_i, C_{2,h} \} = (1 - \lambda^L)\theta$, then both $C_i' < \omega$ and $C_i < \omega$, implying that the resource constraint (21) is slack. Then the CCP can offer the alternative allocation $C_i' = C_i + \varepsilon \frac{p_i}{\alpha}$ and $C_{2,h} = C_{2,h} - \varepsilon$. For $\varepsilon > 0$ arbitrarily small the limited commitment constraint (22) is still satisfied and all other constraints are satisfied by construction. Because $\frac{p_i}{\alpha} < 1$ then the allocation $C_i', C_{2,h}$ uses fewer resources and therefore attains a higher value of the objective. Therefore $C_i' > C_i \geq 0$.

Combining the results in lemmas 7 through 11 and using constraint (23) we then conclude that $C_i = C_i'$. The value of $C_i$ is pinned down by the participation constraint (20). Also, notice that the feasibility constraint (21) is slack. We summarize these findings in the next lemma.

**Lemma 12** The feasibility constraint (20) is always slack and first period consumption for borrowers is

$$C_i = C_i' = \max \left\{ 0, \omega - \frac{p(1 - \lambda^L)\theta}{\alpha} \right\}$$
Proof. Constraint (23) implies $C^H_1 = C^L_1$ and constraint (22) implies $C^H_{2,h} = C^L_{2,h} = (1 - \lambda^L)\theta$. Hence, either both the participation constraint (20) and the feasibility constraint (21) are both binding, or only one is, or $C^i_1 = C^{-i}_1 = 0$ and both constraints are slack.\footnote{There is the possibility that $C^i_1 = 0$ and the participation constraint binds: but this happens if and only if $\omega = (1 - \lambda^L)p\theta$.} Suppose first that the participation constraint (20) is slack and the feasibility constraint (21) binds. Then, the alternative allocation $C^i_1 = C^i_1 = \varepsilon$ for $\varepsilon > 0$ arbitrarily small, is still in the constraint set of problem (P2) but yields higher value of the objective.

Hence, if $C^i_1 > 0$ then either both the participation constraint (20) and the feasibility constraint (21) are binding or only the participation constraint (20) is. Both the participation constraint (20) and the feasibility constraint (21) cannot be binding, however, with $C^{-i}_{2,h} = (1 - \lambda^L)\theta$, $C^i_{2,l} = 0$ and $C^i_1 = C^{-i}_1$. Thus, if $C^i_1 > 0$ only the participation constraint (20) binds and, as a consequence, $C^i_1 = \omega - p(1 - \lambda^L)p\theta$. Alternatively, $C^i_1 = 0$ and the participation constraint (20) is slack (obviously the feasibility constraint (21) is also slack). \[\blacksquare\]

We summarize the result for the CCP problem in the following proposition.

**Proposition 13** A solution to the CCP problem is such that the consumption of the lender is equal across states, $X^H_2 = X^L_2 = X_2$ where $X_2$ is determined by the resource constraint at $t = 2$:

$$X_2 + pC_{2,h} = \omega - C_1 + p\theta$$

So, if:

1. $\omega \leq \frac{(1 - \lambda^L)p\theta}{\alpha}$ then $C_1 = 0$, $C_{2,h} = (1 - \lambda^L)\theta$ and $X_2 = \omega + \lambda_L p\theta$. Let $X_2$ denote such value of $X_2$.

2. $\omega > \frac{(1 - \lambda^L)p\theta}{\alpha}$ then $C_1 = \omega - \frac{(1 - \lambda^L)p\theta}{\alpha}$, $C_{2,h} = (1 - \lambda^L)\theta$ and $X_2 = \frac{(1 - \lambda^L)p\theta}{\alpha} + \lambda_L p\theta$. Let $X_2$ denote such value of $X_2$. \[\blacksquare\]

Notice that $X_2 \geq X_2$.

Therefore the CCP insures lenders against both counterparty default risk and exogenous output realization. It offers the same contract to every borrower: we can interpret this as the CCP setting uniform margin and default fund contributions, regardless of whether borrowers have more or less incentives to deviate, when it cannot observe such incentives. The payments to the borrowers are contingent on the exogenous output realization.

I would take this out: or let’s talk about what we really want to say about the solution to this problem.

There is no special role for the CCP for screening a $H,L$ borrower: there is no efficiency rationale here for screening because borrowers are not more or less
productive depending on their type, they are just different in their temptation to deviate, but the stochastic process for output is the same for both of them. So the equilibrium with central clearing is equivalent to not screening at all but just making sure that nobody deviates assuming they are all the worst type $L$ (they all have the highest temptation to walk out of the contract ex post).

5 Comparing CCP and bilateral clearing solutions

5.1 Low outside option for both borrowers $\frac{\alpha \omega}{p} \leq (1 - \lambda^H) \theta$

If $\frac{\alpha \omega}{p} \leq (1 - \lambda^H) \theta$ then it is also the case that $\frac{\alpha \omega}{p} \leq (1 - \lambda^L) \theta$ since $\lambda^H > \lambda^L$. The solution to the bilateral clearing problem is always such that the limited commitment constraint (LC) is binding and the participation constraint (PC) is slack: however there may be two different solutions for $X^i_2$:

$$X^i_{2h} = \omega + \lambda^i \theta$$
$$X^i_{2l} = \omega$$

if and only if

$$-pu'(\omega + \lambda^i \theta) - (1 - p)u'(\omega) < 0$$
$$\frac{\alpha \omega}{p} \leq [1 - \lambda^i] \theta$$

With respect to the CCP solution ($X^{CCP}_2 = \omega + \lambda L p \theta$) we have that $X^i_{2l} < X^{CCP}_2$ and that $X^i_{2h} > X^{CCP}_2$ for both lenders, since $\lambda_L < \lambda_H$ and $p < 1$.

And

$$X^i_{2h} = \omega - c^*_1 + \lambda^i \theta$$
$$X^i_{2l} = \omega - c^*_1$$

if and only if

$$-pu'(\omega + \lambda^i \theta) - (1 - p)u'(\omega) > 0$$
$$\frac{\alpha \omega}{p} \leq \alpha c^*_1 + [1 - \lambda^i] \theta$$

As when $c_1 = 0$, also when $c_1 > 0$ we have that with respect to the CCP solution $X^i_{2l} < X^{CCP}_2$ and that $X^i_{2h} > X^{CCP}_2$ for both lenders.

Therefore in this case both lenders prefer bilateral clearing ex post in the high state and prefer central clearing ex post in the low state. Ex ante lenders will clear centrally if the insurance they get through the CCP is large enough (or if they are risk averse enough).

For the case where $c_1 = 0$ it this is true if and only if:

$$u(\omega + \lambda L p \theta) > pu(\omega + \lambda^i \theta) + (1 - p) u(\omega)$$
Clearly, if this is satisfied for $\lambda^H$ lenders then it is also satisfied for $\lambda^L$ lenders. In general, lender of type $i$ ex ante prefers to clear centrally rather than bilaterally if and only if $\omega + \lambda^L p^i \theta > X^i_2$, where $X^i_2$ is defined as the certainty equivalent of the utility from bilateral clearing:

$$u \left( X^i_2 \right) = pu \left( \omega + \lambda^i \theta \right) + (1 - p) u \left( \omega \right)$$

For the case where $c^i_1 > 0$ it this is true if and only if:

$$u \left( \omega + \lambda^L p^i \theta \right) > pu \left( \omega - c^i_1 + \lambda^i \theta \right) + (1 - p) u \left( \omega - c^i_1 \right)$$

Similarly we can define the certainty equivalent of the utility from bilateral clearing, $X^i_2$, as:

$$u \left( X^i_2 \right) = pu \left( \omega - c^i_1 + \lambda^i \theta \right) + (1 - p) u \left( \omega - c^i_1 \right)$$

Notice that in this subset of the parameters’ space the limited commitment constraint (LC) is binding and the participation constraint (PC) is slack, therefore the limited commitment friction is strong even though the borrowers’ outside option to trading is not very high so that it is not very expensive for lenders to give them sufficient incentives to deal with the limited commitment friction.

5.2 High outside option for both borrowers $\frac{\alpha \omega}{p} > \left( 1 - \lambda^L \right) \theta$

Central clearing through a CCP implies that both lenders get $X_2 = \frac{(1 - \lambda^L p^i \theta)}{\alpha} + \lambda^L p^i \theta$.

Claim 14 Consider the case where both the limited commitment (LC) and the participation (PC) constraints are binding and $c^i_1 = 0$. Both lenders are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$.

This happens if and only if

$$\frac{(\alpha - p)}{(1 - p)} > \frac{u' \left( \frac{p(1 - \lambda^i \theta)}{\alpha} \right)}{u' \left( \frac{p(1 - \lambda^i \theta)}{\alpha} + \lambda^i \theta \right)}$$

And for lenders we have that

$$X^i_{2h} = \frac{1 - \lambda^i}{\alpha} p^i \theta + \lambda^i \theta$$

$$X^i_{2l} = \frac{1 - \lambda^i}{\alpha} p^i \theta$$

Lenders who are matched with $\lambda^L$ borrowers are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$. 

36
Lenders who are matched with $\lambda^H$ borrowers are ex post better off with bilateral clearing if the realization of output is $h$ if and only if

$$\lambda^H - \lambda_L p > \frac{(\lambda^H - \lambda_L) p}{\alpha}$$

$$(\lambda^H - \lambda_L) p + (1 - p) \lambda^H > \frac{(\lambda^H - \lambda_L) p}{\alpha}$$

$$(\lambda^H - \lambda_L) p \left(1 - \frac{1}{\alpha}\right) + (1 - p) \lambda^H > 0$$

which is always true. Whereas they are ex post better off with central clearing if the realization of output is $l$.

Therefore both lenders are ex post better off with bilateral clearing if the realization of output is $h$ whereas are ex post better off with central clearing if the realization of output is $l$.

As above we can define the certainty equivalent of the utility from bilateral clearing, $X^i_2$, as:

$$u \left( X^i_2 \right) = p u \left( \frac{1 - \lambda^i}{\alpha} p + \lambda^i \theta \right) + (1 - p) u \left( \frac{1 - \lambda^i}{\alpha} p \theta \right)$$

so that central clearing is preferred by a lender matched with a borrower with type $\lambda^i$ if and only if

$$\frac{(1 - \lambda_L) p \theta}{\alpha} + \lambda_L p \theta > X^i_2$$

Claim 15 Consider the case where the limited commitment constraint (LC) is slack and the participation constraint (PC) is binding and $c_1 = 0$

This happens if and only if

$$\theta > \frac{\alpha \omega}{p} \geq (1 - \lambda^i) \theta \quad (24)$$

$$\frac{u' \left( \omega \right)}{u' \left( \omega + \theta - \frac{\alpha \omega}{p} \right)} > \frac{(\alpha - p)}{(1 - p)} \quad (25)$$

And for lenders we have that:

$$X^i_{2h} = \omega + \theta - \frac{\alpha \omega}{p}$$

$$X^i_{2l} = \omega$$

Recall that in this subset of the parameters’ space we have that $\frac{\alpha \omega}{p} > (1 - \lambda^L) \theta$.

Since $X^i_{2h}, X^i_{2l}$ are independent of $i$ then both lenders who are matched with $\lambda^L$ borrowers and lenders who are matched with $\lambda^H$ borrowers have the same incentives to clear bilaterally or centrally.
Therefore lenders are ex post better off with bilateral clearing if the realization of output is $h$ if and only if

$$\frac{\theta \left(1 - \left[\frac{(1 - \lambda_L)}{\alpha} + \lambda_L\right] p\right)}{\left(\frac{\alpha}{p} - 1\right)} > \omega$$

$$\frac{\lambda L p \theta}{\alpha} \left(\alpha - p\right) - \left(\alpha - 1\right) p \lambda L > \omega$$

(26)

Whereas they are ex post better off with central clearing if the realization of output is $l$ if and only if

$$\omega - \frac{(1 - \lambda_L)}{\alpha} p \theta \omega < \lambda L p \theta$$

$$\frac{\alpha \omega}{p} < \left[\alpha \lambda_L + (1 - \lambda_L)\right] \theta$$

(27)

Recall that this is the case where the LC is slack but still $c_1 = 0$ despite $\alpha > 1$ because the probability of the good outcome is relatively small: in bilateral clearing the only way to guarantee some high consumption to the lender in the low realization is to save, that is equivalent to paying the borrower less $c_1$. In this case obtaining insurance against the realization of output in the bilateral clearing setup is more costly because the tool of using $c_1$ in order to give the borrower incentives needs to be employed. In the bilateral clearing case the problem is that the best instrument to provide incentives (collateral) is also the only instrument to provide insurance across states (storage). So this is a case where central clearing could be useful in any state (if $p$ is not too low, that is to say the probability of success of output, which is the total measure of additional output in the high state in the CCP clearing, because of full diversification of the output realization shock. So if there is not much to gain there, in terms of resources that can be used to provide insurance, there is no incentive to clear centrally). Or in other words, if we want to interpret the necessary and sufficient conditions for bilateral vs central clearing, (26) and (27), in terms of upper bounds on $\omega$, then central clearing is better unambiguously any time the borrower is not endowed with a lot of consumption good. This is so because in bilateral clearing if $\omega$ is large then there are relatively a lot of resources to store from one period to the next so that having to forgo the provision of incentives via $c_1$ is not too costly. With central clearing instead, the participation constraint always binds, that is to say lenders always pay the borrowers their present discounted value of the outside option: when this is relatively large then the benefits from insurance through a CCP - coming from diversification (getting a lot of $\theta$ regardless of the state each lender is in) rather than having to store more goods - are relatively small.

Also notice the upper bounds on $\omega$ needed in this case for the CCP to be always preferred even ex post, does not tighten the necessary and sufficient conditions (24) and (25) for the bilateral solution. In fact (24) is already taken
into account, and if (25) is satisfied at a given $\omega$ then it is always satisfied at any $\omega \leq \omega$.

Clearly the opposite is also true: it doesn’t matter how risk averse lenders are, they may always prefer to clear bilaterally if $\omega > \omega = \max (\omega_1, \omega_2)$ where $\omega_1 = \frac{p\theta (\alpha - p) - (\alpha - 1)p\lambda_L}{\alpha - p}$, $\omega_2 = \frac{p\theta}{\alpha} [\alpha\lambda_L + (1 - \lambda_L)]$ defined by (26) and (27).

Claim 16 Consider the case where the limited commitment constraint (LC) is slack and the participation constraint (PC) is binding and $c_i^1 > 0$.

This happens if and only if

$$\theta > \frac{\alpha (\omega - c_{13})}{p} \geq (1 - \lambda^i) \theta$$

$$\frac{u'(\omega)}{u'(\omega + \theta - \frac{\alpha\omega}{p})} < \frac{(\alpha - p)}{(1 - p)}$$

(28)

(29)

And for lenders we have that:

$X_{2h}^i = \omega - c_{13} + \theta - \frac{\alpha (\omega - c_{13})}{p}$

$X_{2l}^i = \omega - c_{13}$

where $c_{13}$ solves

$$(1 - p)u'(\omega - c_{13}) = (\alpha - p) u'(\omega - c_{13} + \theta - \frac{\alpha (\omega - c_{13})}{p})$$

Since $X_{2h}^i, X_{2l}^i$ are independent of $i$ then both lenders who are matched with $\lambda^L$ borrowers and lenders who are matched with $\lambda^H$ borrowers have the same incentives to clear bilaterally or centrally. In this case however things are different than with $c_1 = 0$ and are more interesting since it becomes apparent that the schedule of payments matters: in state $l$ the central clearing solution is preferred ex post, but in state $h$ the bilateral solution is ex post preferred by both lenders if and only if

$$-c_{13} - \frac{\alpha (\omega - c_{13})}{p} > \frac{\alpha\omega}{p}$$

$$c_{13} \left(1 - \frac{\alpha}{p}\right) < 0$$

which is always true.

Notice that with respect to the previous case the present value of consumption to the borrower is the same, because the participation constraint is binding, and also the insurance purpose of $c_1$ is still at play here because the limited commitment constraint is slack and $c_{13}$ is determined by the marginal rate of substitution between states. However lenders are optimally using some $c_1$ to provide incentives, which is a cheaper way to do so rather than using $c_{2h}$. This
implies that in state \( h \) lenders can consume more \( c_2 h \) which makes them prefer ex post the bilateral clearing solution.

Also notice that this solution arises when lenders are not very risk averse (i.e. \( (29) \) holds rather than \( (25) \)).

Ex ante lenders prefer the CCP clearing arrangement if and only if \( \frac{(1-\lambda L)}{\alpha} + \lambda L \) where \( X_2 \) is defined as the certainty equivalent of the utility from bilateral clearing:

\[
u(X_2) = pu\left(\omega - c_{13} + \theta - \frac{\alpha(\omega - c_{13})}{p}\right) + (1 - p) u(\omega - c_{13})
\]

6 Other CCP contracts

6.1 Contract with screening and borrowers’ separation

Under the assumption that the CCP cannot observe whether each lender has screened their borrower (and obviously cannot observe the outcome of the screening either) then the CCP has to induce lenders to screen and report their borrower’s type truthfully. If this is feasible then the CCP has full information about borrowers’ types: thus let \( m^i_L \in \{\lambda^H, \lambda^L\} \) denote the message sent to the CCP by a lender matched with borrower \( i \), whose type is \( \lambda^i \). A feasible contract for the CCP is to set:

\[
X_2 (m^i_L, \lambda^i) = \begin{cases} 
X^{FI,i}_2 & \text{if } m^i_L = \lambda^i \\
0 & \text{if } m^i_L \neq \lambda^i 
\end{cases}
\]

Let \( V^{FI} \) denote the ex ante full information payoff to the lender, before he knows whether his borrower’s type is \( \lambda^H \) or \( \lambda^L \) and consider the following

\[
(P0^{FI}) \quad V^{FI} = \max_{qu\left(X^{H}_2\right) + (1 - q) u\left(X^{L}_2\right)} \\
\text{s.t. } \alpha C^i_1 + pC^i_{2h} + (1 - q) C^i_{2l} \geq \alpha \omega, \forall i \quad (30) \\
\quad C^i_{2h} \geq (1 - \lambda^i) \theta, \forall i \quad (31) \\
\quad 0 \leq C^i_1 \leq \omega, \forall i \quad (32) \\
\quad qX^H_2 + (1 - q) X^L_2 + qpC^H_{2h} + (1 - q) C^L_{2l} + q(1 - p) C^H_{2l} + (1 - q) (1 - p) C^L_{2l} \leq \omega - qC^H_1 - (1 - q) C^L_1 + p\theta \quad (33) \\
\quad -\gamma + qu\left(X^{H}_2\right) + (1 - q) u\left(X^{L}_2\right) \geq \max\left(u\left(X^L_2\right), (q + (1 - q) (1 - p)) u\left(X^H_2\right) + (1 - q) pu(0)\right) \quad (34)
\]

where the last constraint is an incentive constraint for the lender: lenders can deviate by not screening and reporting \( \lambda^L \), in which case they always consume \( u\left(X^L_2\right) \), or by reporting \( \lambda^H \), in which case with probability \( q \) the deviation is not detected by the CCP because the borrower is actually a \( \lambda^H \) type, and with probability \( (1 - q) (1 - p) \) the deviation is not detected because despite...
the borrower being a $\lambda^L$ type, his output realization is low. Therefore with probability $(1-q)p$ the deviation is detected by the CCP and lenders consume nothing, because the borrower is a $\lambda^L$ type and the output realization is high so that he steals $(1-\lambda^L)\theta > (1-\lambda^H)\theta$.

Notice that, the incentive constraint (34) is an ex-ante incentive constraint: it guarantees that a lender has an incentive to pay the screening cost rather than making a report about his borrower without actually knowing his type. Besides incentive constraint (34) the CCP faces an ex-post incentive constraint that gives the lender incentives to report his borrower’s type truthfully: a lender who finds out that his borrower is a $\lambda$ type could lie and report a type $\lambda^H$ instead. If that happened the CCP would find that out at $t=2$ when the $\lambda$ borrower would consume $(1-\lambda^L)\theta$ if that exceeds the consumption the payment received by the CCP, $C^H_{2t}$. Thus the CCP could punish the lender by not paying him at all, and the lender’s payoff would be $u(0)$. The opposite case, of a lender reporting a $\lambda^L$ type when he finds out his type is $\lambda^H$, would never be detected by the CCP: this is because the payment to this borrower would have to satisfy the limited commitment constraint (31) for $i=L$, which implies that the limited commitment constraint (31) for $i=H$ is also satisfied. Thus, in order to have lenders truthfully report the type of borrower they screened, the CCP faces also the following ex-post incentive constraint $u(X^H_i) \geq u(X^L_i)$. This constraint, however, is always satisfied by any allocation that satisfies (34). Thus it is redundant and we do not consider it in (P0FI).

Also, notice that the incentive constraint (34) generates a non-convex set of allocations the CCP can choose. In that case, let us then define $w^H = u(X^H_i)$ and rewrite the problem with the CCP choosing $\{w, C^L_i, C^H_{2i}\}_{i=H,L,s=h,l}$:

$$\hat{P}^{FI} \quad V^{FI} = \max \quad qw^H + (1-q)w^L$$

s.t. $\alpha C^i_1 + pC^i_{2h} + (1-p)C^i_{2l} \geq \alpha \omega, \forall i$ (35)

$C^i_{2h} \geq (1-\lambda^L)\theta, \forall i$ (36)

$C^i_1 \leq \omega, \forall i$ (37)

$qw^{-1}(w^H) + (1-q)u^{-1}(w^H) + qpC^H_{2h} + (1-q)pC^L_{2h} +$ \[+q(1-p)C^H_{2l} + (1-q)(1-p)C^L_{2l} \leq \omega - qC^H_1 - (1-q)L^i_1 + p\theta \] (38)

$-\gamma + qw^H + (1-q)w^L \geq$ \[\max (w^L, (q + (1-q)(1-p))w^H + (1-q)pu(0)) \] (39)

Lemma 17 $\alpha > 1$ implies that a solution to (PFI) is such that i) (37) (i.e. the resource constraint at $t=1$) is slack; ii) $C^2_{2i} = 0$ for all $i$; iii) (38) (i.e. the resource constraint at $t=2$) binds; iv) $C^i_1 > 0$ if and only if the participation constraint (35) is binding for borrower $i = L, H$, or $\omega = \frac{(1-\lambda^L)\theta}{\alpha}$; v) the limited commitment constraint (36) is always binding for both borrowers $i = L, H$; vi) the incentive constraint (39) is always binding.

Proof.
i) Suppose not. Then $C_1^i = \omega$ for at least $i = L$ or $i = H$. Without loss of generality suppose $i = L$, the other case is analogous. Because the limited commitment constraint \[ 36 \] must be satisfied then $C_{2,h}^L \geq (1 - \lambda^L)\theta > 0$ implying $\alpha C_1^L + pC_{2,h}^L > \alpha\omega$. Thus the participation constraint \[ 35 \] is slack: then an alternative allocation can be constructed such that: \[ 35 \] is still satisfied; \[ 35 \] is such that $\hat{\lambda}^L$ with $\hat{\gamma}$ because by arbitrariness small. Because \[ 1t \] it follows that \[ 1t \] follows: \[ 1t \] and \[ 1t \] The allocation $\hat{\lambda}^L = \hat{\lambda}^L_{1} + \hat{\lambda}^L_{2}, \hat{\gamma} = \hat{\gamma}_{1}^L + \hat{\gamma}_{2}^L$ for both $i = L$ and $i = H$, if $\hat{u}(X^L) = (q + (1 - q)(1 - p))u(\hat{X}^H) + (1 - q)pu(0)$. This guarantees that the resource constraint at $t = 2$, \[ 38 \] is still satisfied and that $w^H$ and $w^L$ increase by the same amount so that the incentive constraint \[ 39 \], in this case simply $q(w^H - w^L) \geq \gamma$, is still satisfied. Additionally, if $u(X^L_{1}) < (q + (1 - q)(1 - p))u(\hat{X}^H_{2}) + (1 - q)pu(0)$, then $\hat{X}^L_{1} = X^L_{1} + (1 - q)\epsilon^L_{1}$ for $i = L, H$, where $\epsilon^H_{1}$ is such that $\hat{w}^H = u(\hat{X}^H_{2}) = w^H + u(\epsilon^H_{1})$ and $\epsilon^L_{1}$ is such that $\hat{w}^L = u(X^L_{2}) = w^L + \epsilon^L$. By construction this allocation is still in the constraint set to problem $(\hat{P}^{FI})$ and yields strictly higher value of the objective. Thus $C_1^i < \omega, \forall i$.

ii) Suppose not. Then, without loss of generality suppose $C_{2,i}^L > 0$, since the other case is analogous. Then there exists an alternative allocation defined as follows: $\hat{C}_{2,i}^L = C_{2,i}^L - \epsilon, \hat{C}_i = C_i^L + \frac{(1 - p)L}{\alpha}, \hat{X}_{2,i}^L = X_{2,i}^L + \epsilon(1 - \frac{(1 - p)L}{\alpha})$. By construction the participation constraint \[ 35 \], the resource constraint at $t = 1$ \[ 37 \] and the resource constraint at $t = 2$ \[ 38 \] are still satisfied for $\epsilon > 0$ arbitrarily small. Because $\frac{(1 - p)L}{\alpha} < 1$ then $\hat{X}_{2,i}^L > X_{2,i}^L$. Thus allocation $\hat{C}_{2,i}^L, \hat{C}_i, \hat{X}_{2,i}^L$ is feasible and yields strictly higher value of the objective in $(\hat{P}^{FI})$. Thus $C_{2,i}^L = 0$.

iii) Suppose not. Then there exists an alternative allocation defined by $\hat{X}^L_{1} = X^L_{1} + \epsilon^L_{1}$ for both $i = L$ and $i = H$ defined similarly to i) in order to satisfy the incentive constraint \[ 39 \] in either case. This allocation is still in the constraint set to problem $(\hat{P}^{FI})$ and yields strictly higher value of the objective. Thus \[ 33 \] must bind at a solution to $(\hat{P}^{FI})$.

iv) By contradiction. Suppose that $C_1^i > 0$ and the participation constraint for borrower $i$ is slack. Without loss of generality suppose that $i = L$: the case for $i = H$ is analogous. Then there exists an alternative allocation defined as follows: $\hat{C}_1^L = C_1^L - \epsilon$ and $X^L_{2} = X^L_{2} + \epsilon^L_{2}$ for $i = L, H$ with $\epsilon^L_{2}$ as specified in case i) to satisfy the incentive constraint \[ 39 \]. Then the allocation $\hat{C}_1^L, X^L_{2}$ is still in the constraint set of problem $(\hat{P}^{FI})$ and yields higher value of the objective. Thus $C_1^L > 0$ implies \[ 35 \] binds. Suppose now that the participation constraint for borrower $L$ is binding and $C_1^L = 0$: then either $\omega = \frac{(1 - \lambda^L)p\theta}{\alpha}$ or $C_{2,h}^L = \frac{\alpha\omega}{p}$, because by ii) it follows that $C_{2,i}^L = 0$ for all $i$. Suppose then that $\omega > \frac{(1 - \lambda^L)p\theta}{\alpha}$ and $C_{2,h}^L = \frac{\alpha\omega}{p}$. If $C_{2,h}^L$ is part of a solution to $(\hat{P}^{FI})$ then $C_{2,h}^L \geq (1 - \lambda^L)\theta$. Then consider the alternative allocation defined by $\hat{C}_1^L = \epsilon > 0, \hat{C}_{2,h}^L = C_{2,h}^L - \frac{\alpha\epsilon}{p}$, with $\frac{\alpha}{p} > 1$, and $\hat{X}^L_{2} = X^L_{2} + (1 - q)(\frac{\alpha}{p} - 1)\epsilon^L_{2}$ for $i = L, H$ with $\epsilon^L_{2}$ defined similarly to case i) in order to satisfy the incentive constraint \[ 39 \]. Then this allocation is still in the constraint set to problem $(\hat{P}^{FI})$ but yields higher value
of the objective.

v) By contradiction. Suppose that the limited commitment constraint for \( i = L \) is slack, the case for \( i = H \) is analogous. Then consider the alternative allocation \( \hat{C}^{L}_{2,h} = C^{L}_{2,h} - \varepsilon, \hat{C}^{L}_{1} = C^{L}_{1} + \frac{p\varepsilon}{\alpha} \) which still satisfies the participation constraint (35), and because \( \frac{p}{\alpha} < 1 \) then let \( \hat{X}^{L}_{2} = X^{L}_{2} + \varepsilon'(1 - \frac{p}{\alpha}) \) for \( i = L, H \) be defined similarly to case i) to satisfy the incentive constraint (39). Then this allocation is still in the constraint set to problem \( (P^{FI}) \) but yields higher value of the objective.

vi) By contradiction. Suppose that (39) is slack. Then a solution must be such that \( w^{H} = w^{L} \); if not then \( w^{H} > w^{L} \) (the other case where \( w^{H} < w^{L} \) is analogous). Then there exist an alternative allocation such that \( \hat{w}^{H} = w^{H} - \varepsilon^{H} \) and \( \hat{w}^{L} = w^{L} + \varepsilon^{L} \) where \( \varepsilon^{i} \) is defined so that the resource constraint at \( t = 2 \), (38) is still satisfied. That is to say \( qu^{-1}(\hat{w}^{H}) + (1 - q)u^{-1}(\hat{w}^{L}) = qu^{-1}(w^{H}) + (1 - q)u^{-1}(w^{L}) \). This allocation is then in the constraint set to problem \( (P^{FI}) \) and yields strictly higher value of the objective. Thus \( w^{H} = w^{L} \). But this violates (39). Notice that lemma 17 implies that the CCP never chooses to hold collateral (i.e. pay smaller \( C^{i}_{1} \)) in order to provide insurance to lenders against the borrower’s output shock. This happened in the problem with bilateral clearing because the lender needs to self insure against that shock, but the CCP can pool idiosyncratic risks among all borrowers. Thus insurance against \( \tilde{\theta} \) can always be achieved in a CCP with no need to hold collateral. Collateral in a CCP will always be used only to provide incentives to borrowers to participate to the contract.

6.1.1 Incentive constraint -Case 1:

\[
 u \left(X^{L}_{2}\right) > (q + (1 - q)(1 - p)) u \left(X^{H}_{2}\right) + (1 - q) pu(0) \quad (40)
\]

This is equivalent to

\[
\max \left(w^{L}, (q + (1 - q)(1 - p)) w^{H} + (1 - q) u(0)\right) = w^{L} \quad (41)
\]

So that the incentive constraint can be rewritten as:

\[
q \left(w^{H} - w^{L}\right) \geq \gamma \quad (42)
\]

Then the CCP chooses \( \{w^{i}, C^{i}_{1}, C^{i}_{2h}\}_{i=H,L,s=h,l} \) to solve:

\[
(P^{FI}) \quad V^{FI} = \max_{s.t.} qu^{H} + (1 - q) w^{L} \quad (43)
\]

\[
\alpha C^{i}_{1} + p C^{i}_{2h} \geq \alpha \omega, \forall i \quad (44)
\]

\[
C^{i}_{2h} \geq (1 - \lambda^{i}) \theta, \forall i \quad (45)
\]

\[
C^{i}_{1} \leq \omega, \forall i \quad (46)
\]

\[
qu^{-1}(w^{H}) + (1 - q) u^{-1}(w^{L}) + gpC^{H}_{2h} + (1 - q) pC^{L}_{2h} \leq \omega - qC^{i}_{1} - (1 - q) C^{i}_{2h} + p\theta \quad (47)
\]

43
\[ q \left( w^H - w^L \right) \geq \gamma \]

(48)

where \( \mu^t, \eta^t, \beta, \beta_{tC} \) denote Lagrange multipliers on the participation constraint (44), limited commitment constraint (45), resource constraints at \( t = 1 \) (46) and at \( t = 2 \) (47), and incentive constraint (48).

Notice that because \( u \) is strictly concave then \( u^{-1} \) is strictly convex, thus the resource constraint is still a convex set. Then, the optimal choice of \( \{ w^i, C^t_1, C^t_2 \}_{i=H, L, s= h, l} \) is such that:

\[
q \left( 1 - \beta u^{-1'} (w^H) + \beta_{tC} \right) = 0 \\
(1 - q) \left( 1 - \beta u^{-1'} (w^L) - \beta_{tC} \right) = 0
\]

\[
\alpha \mu^H - q \beta \leq 0 \quad \text{if } C^H_1 > 0 \\
\alpha \mu^L - (1 - q) \beta \leq 0 \quad \text{if } C^L_1 > 0 \\
\eta^H (p \mu^H + q \beta) = 0 \\
\eta^L (p \mu^L + (1 - q) p \beta) = 0
\]

In the CCP problem, \( C^t_1 > 0 \) if and only if the participation constraint for borrower \( i \) binds. Also, \( \beta_{tC} > 0 \): if not, then it is easy to show that a solution is such that \( w^H = w^L \): as a consequence the incentive constraint is violated. Finally, the limited commitment constraint is always binding. Therefore:

**Participation constraint binds \( \forall i \)**

Then: \( C^t_1 > 0, \forall i \) and \( \mu^t > 0, \forall i, \) and necessary and sufficient conditions for a solution to (P1FL) are:

\[
\mu^H = \frac{q \beta}{\alpha} (1 - q) \beta \\
\mu^L = \frac{q \beta}{\alpha}
\]

\(^{27}\)Indeed consider \( \left( \tilde{w}^i, \tilde{C}^i_1, \tilde{C}^i_2 \right), \left( \hat{w}^i, \hat{C}^i_1, \hat{C}^i_2 \right) \) for which the resource constraint at \( t = 2 \) is satisfied, and let \( s^i = \delta \hat{s}^i + (1 - \delta) (\tilde{s}^i) \) for \( s^i = w^i, C^i_1, C^i_2 \). The resource constraint evaluated at \( w^h, C^h_1, C^h_2 \) becomes:

\[
qu^{-1} (w^H) + (1 - q) u^{-1} (w^L) \leq \omega - q \left( C^H_1 + p C^H_2 \right) + (1 - q) \left( C^L_1 - p C^L_2 \right) + \rho \theta
\]

Because \( u^{-1} \) is strictly convex:

\[
qu^{-1} (w^H) + (1 - q) u^{-1} (w^L) < \delta \left[ qu^{-1} (\tilde{w}^H) + (1 - q) u^{-1} (\tilde{w}^L) \right] + (1 - \delta) \left[ qu^{-1} (\hat{w}^H) + (1 - q) u^{-1} (\hat{w}^L) \right]
\]

\[
< \delta \left[ \omega - q \left( \tilde{C}^H_1 + p \tilde{C}^H_2 \right) - (1 - q) \left( \tilde{C}^L_1 + p \tilde{C}^L_2 \right) + \rho \theta \right] + (1 - \delta) \left[ \omega - q \left( \hat{C}^H_1 + p \hat{C}^H_2 \right) - (1 - q) \left( \hat{C}^L_1 + p \hat{C}^L_2 \right) + \rho \theta \right]
\]

\[
= \omega - q \left( \tilde{C}^H_1 + p \tilde{C}^H_2 \right) - (1 - q) \left( \tilde{C}^L_1 - p \tilde{C}^L_2 \right) + \rho \theta
\]
\[ \eta^H = qp\beta - \mu^H = qp\beta \left(1 - \frac{1}{\alpha}\right) \]

\[ \eta^L = (1 - q) p\beta - \mu^L = (1 - q) p\beta \left(1 - \frac{1}{\alpha}\right) \]

Also, lemma [17] implies that the incentive constraint of the lender, (39), and the limited commitment constraint, (36), bind: therefore \( w^H = w^L + \frac{\gamma}{q} \) and \( C_{2h}^i = (1 - \lambda^i) \theta, \forall i \). The participation constraint binds by assumption, so \( \alpha C_1^i + p C_{2h}^i = \alpha \omega, \forall i \). From the resource constraint at \( t = 2 \) we then have:

\[
qu^{-1} \left(w^L + \frac{\gamma}{q}\right) + (1 - q) u^{-1}(w^L) + qp(1 - \lambda^H)\theta + (1 - q) p(1 - \lambda^L)\theta = q \left[p \frac{(1 - \lambda^H)\theta}{\alpha}\right] + (1 - q) \left[p \frac{(1 - \lambda^L)\theta}{\alpha}\right] + \rho \theta \tag{49}\]

which gives the equilibrium payoff \( w^L \). This equilibrium exists if and only if:

1. \( \frac{\alpha \omega}{\rho} > (1 - \lambda^L) \theta \), otherwise the participation constraint would not bind for both borrower types.

2. equation (49) has a solution, that is if there exists \( w^L \geq 0 \) such that:

\[
qu^{-1} \left(w^L + \frac{\gamma}{q}\right) + (1 - q) u^{-1}(w^L) = p \theta \left[1 - \left(q \left(1 - \lambda^H \right) \left(1 - \frac{1}{\alpha}\right) + (1 - q) \left(1 - \lambda^L \right) \left(1 - \frac{1}{\alpha}\right) \right]\right] = p \theta \left[\frac{1}{\alpha} + (q \lambda^H + (1 - q) \lambda^L) \left(1 - \frac{1}{\alpha}\right) \right]

If \( u(0) = 0 \) and if \( u \) is bounded then there exists such \( w^L \in [0, \infty) \) and it is unique, if and only if:

\[
\frac{\gamma}{q} < u \left(\frac{p \theta}{q} \left[\frac{1}{\alpha} + (q \lambda^H + (1 - q) \lambda^L) \left(1 - \frac{1}{\alpha}\right) \right]\right)
\]

3. lenders’ utility \( w^L \), solving (49), together with \( w^H \) derived from the incentive constraint (42), are such that (40) is satisfied. Thus, the relevant case for the incentive constraint (39) is indeed case 1, and the incentive constraint is (42).

**Participation constraint binds only for \( i = H \)**

Consider now the case where \( C_1^i = 0 \) for one \( i \) and the relevant participation constraint is slack. Because \( (1 - \lambda^L) \theta > (1 - \lambda^H) \theta \), it must be that it is the participation constraint for a \( \lambda^L \) borrower to be slack. Then we have: \( C_1^i > 0 = C_1^H C_{2h}^i = (1 - \lambda^i) \theta, \forall i, \alpha C_1^i + p (1 - \lambda^H) \theta = \alpha \omega \), and still \( w^H = w^L + \frac{\gamma}{q} \).

From the resource constraint at \( t = 2 \) we then have:

\[
qu^{-1} \left(w^L + \frac{\gamma}{q}\right) + (1 - q) u^{-1}(w^L) + qp(1 - \lambda^H)\theta + (1 - q) p(1 - \lambda^L)\theta = \]

45
which gives the equilibrium payoff $w^L$. This is an equilibrium if and only if:

1. \((1 - \lambda^L) \theta > \frac{\alpha \omega}{p} > (1 - \lambda^H) \theta\), which verifies $C_1^H > 0$ and $C_1^L = 0$.

2. equation (50) has a solution, that is if there exists $w^L \geq 0$ such that:

   \[
   qu^{-1} \left( w^L + \frac{\gamma}{q} \right) + (1 - q) u^{-1} (w^L) = (1 - q) \omega + p\theta \left[ 1 - q (1 - \lambda^H) \left( \frac{1 - 1}{\alpha} \right) - (1 - q) (1 - \lambda^L) \right] \\
   = (1 - q) \omega + p\theta \left[ q\lambda^H + \frac{q(1 - \lambda^H)}{\alpha} + (1 - q) \lambda^L \right]
   \]

   If $u(0) = 0$ and $u$ is bounded, then there exists such $w^L \in [0, \infty)$ and it is unique if and only if:

   \[
   \frac{\gamma}{q} < u \left( \frac{1 - q}{q} \omega + p\theta \left[ \lambda^H + \frac{(1 - \lambda^H)}{\alpha} + \frac{(1 - q)}{q} \lambda^L \right] \right)
   \]

3. lenders’ utility $w^L$, solving (50), together with $w^H$ derived from the incentive constraint (42), are such that (40) is satisfied. Thus, the relevant case for the incentive constraint (39) is indeed case 1, and the incentive constraint is (42).

**Participation constraint slack $\forall i$**

Consider finally the case where $C_1^i = 0$, $\forall i$ and the participation constraint is slack for both types. As in the previous cases we have the limited commitment constraint (36) binding for $i = L, H$: $C_{2h}^i = (1 - \lambda^i) \theta \forall i$, and still lenders’ incentive constraint (39) is binding: $w^H = w^L + \frac{\gamma}{q}$. From the resource constraint at $t = 2$ we then have:

\[
qu^{-1} \left( w^L + \frac{\gamma}{q} \right) + (1 - q) u^{-1} (w^L) + qp(1 - \lambda^H)\theta + (1 - q) p(1 - \lambda^L)\theta = \omega + p\theta
\]

which gives the equilibrium payoff $w^L$.

This equilibrium then exists if an only if

1. \((1 - \lambda^H) \theta > \frac{\alpha \omega}{p}\), otherwise $C_1^H > 0$ and $C_1^L > 0$ since \((1 - \lambda^L) \theta > (1 - \lambda^H) \theta\).

2. equation (51) has a solution, that is if there exists $w^L \geq 0$ such that:

   \[
   qu^{-1} \left( w^L + \frac{\gamma}{q} \right) + (1 - q) u^{-1} (w^L) = \omega + p\theta \left[ 1 - (q (1 - \lambda^H) + (1 - q) (1 - \lambda^L)) \right] \\
   = \omega + p\theta \left[ q\lambda^H + (1 - q) \lambda^L \right]
   \]
If \( u(0) = 0 \) and \( u \) is bounded then there exists such \( w^L \in [0, \infty) \) and it is unique if and only if:

\[
\frac{\gamma}{q} < u\left( \frac{\omega}{q} + p \theta \left[ \lambda^H + \frac{(1-q)}{q} \lambda^L \right] \right)
\]

3. lenders’ utility \( w^L \), solving (51), together with \( w^H \) derived from the incentive constraint (42), are such that (40) is satisfied. Thus, the relevant case for the incentive constraint (39) is indeed case 1, and the incentive constraint is (42).

### Comments
Notice that we are constructing an equilibrium where the CCP obtains truthful reports about the borrowers’ types from each lender, and, as a consequence, offers each borrower a contract \( (C_1^i, C_2^i) \) that satisfies his limited commitment and participation constraint. Therefore each borrower accepts that contract. If we were trying to have the CCP elicit the borrower’s type directly, without having lenders incur the screening costs, the CCP would have to offer borrowers a menu of contracts such that each borrower will self select into the contract designed for his type.

6.1.2 Incentive constraint -Case 2:

\[ u \left( X^L_2 \right) < (q + (1 - q)(1 - p)) u \left( X^H_2 \right) + (1 - q) pu(0) \]  

(52)

The incentive constraint (34) can be rewritten as

\[ p \left[ u \left( X^H_2 \right) - u(0) \right] \geq u \left( X^H_2 \right) - u \left( X^L_2 \right) + \frac{\gamma}{(1-q)} \]  

(53)

Then the CCP chooses \( \{w^i, C_1^i, C_2^i\}_{i=H,L,s=h,l} \) to solve:

\[
(V^{FI}) \quad V^{FI} = \max_{w^H, w^L} qw^H + (1 - q) w^L
\]

s.t.

\[
\begin{align*}
\alpha C_1^i + p C_2^i & \geq \alpha \omega, \forall i \\
C_2^i & \geq (1 - \lambda^i) \theta, \forall i \\
C_1^i & \leq \omega, \forall i \\
qu^{-1} (w^H) + (1 - q) u^{-1} (w^L) + qp C_2^H + (1 - q) p C_2^L & \leq \omega - q C_1^H - (1 - q) C_1^L + p \theta \\
u \left( X^L_2 \right) - (1 - p) u \left( X^H_2 \right) - pu(0) & \geq \frac{\gamma}{(1-q)}
\end{align*}
\]

(54) - (59)

where \( \mu^i, \eta^i, \beta, \beta^I \) denote Lagrange multipliers on the participation constraint (55), limited commitment constraint (56), resource constraints at \( t = 1 \) (57) and at \( t = 2 \) (58), and incentive constraint (59).

As in the CCP problem \((P^{FI})\) we can rewrite this problem in terms of lender’s utility levels: \( w^H = u(X^H_2) \) and \( w^L = u(X^L_2) \), and the results of lemma 17 carry through. Therefore, we can simply analyze the cases where the participation constraint binds or is slack for \( i = L, H \).
Participation constraint binds \( \forall i \)

Then \( C_{i}^{1} > 0, \forall i; \) the incentive constraint (59) binds: \( u(X_{i}^{L}) = (1 - p) u(X_{i}^{H}) + pu(0) + \frac{\gamma}{(1 - q)}; \) the participation constraints (55) bind for both \( i = L, H; \alpha C_{i}^{1} + pC_{i}^{2} = \omega, \forall i; \) the limited commitment constraints (56) bind for both \( i = L, H; \quad C_{i}^{1} = (1 - \lambda^{i}) \theta, \forall i. \) From the resource constraint at \( t = 2 \) and using (55) and (59) we then have:

\[
qu^{-1}(w^{H}) + (1 - q)u^{-1}
\begin{align*}
&\left( (1 - p) w^{H} + pu(0) + \frac{\gamma}{(1 - q)} \right) \\
&= \frac{p(1 - \lambda^{H}) \theta}{\alpha} + (1 - q)\frac{p(1 - \lambda^{L}) \theta}{\alpha} + p\theta[q \lambda^{H} + (1 - q) \lambda^{L}] \\
&= p\theta[q \left( \frac{\lambda^{H} + (1 - \lambda^{H})}{\alpha} \right) + (1 - q) \left( \frac{\lambda^{L} + (1 - \lambda^{L})}{\alpha} \right)]
\end{align*}
\]

which gives the lender’s equilibrium utility if matched with a \( \lambda^{H} \)-borrower: \( w^{H}. \)

This equilibrium then exists if an only if

1. equation (60) has a solution, that is if there exists \( w^{H} \in [0, \infty) \) such that:

\[
q u^{-1}(w^{H}) = p\theta \left[ q \left( \frac{\lambda^{H} + (1 - \lambda^{H})}{\alpha} \right) + (1 - q) \left( \frac{\lambda^{L} + (1 - \lambda^{L})}{\alpha} \right) \right] - (1 - q)u^{-1}
\begin{align*}
&\left( (1 - p) w^{H} + pu(0) + \frac{\gamma}{(1 - q)} \right)
\end{align*}
\]

If \( u(0) = 0 \) and \( u \) is bounded then there exists such \( w^{H} \in [0, \infty) \) and it is unique if and only if:

\[
\frac{\gamma}{1 - q} < u \left( \frac{p\theta}{(1 - q)} \left[ q \left( \frac{\lambda^{H} + (1 - \lambda^{H})}{\alpha} \right) + (1 - q) \left( \frac{\lambda^{L} + (1 - \lambda^{L})}{\alpha} \right) \right] \right)
\]

2. \( \frac{\omega}{p} > (1 - \lambda^{L}) \theta, \) otherwise \( C_{i}^{L} > 0 \) cannot be part of a solution to \((P2FI).\)

3. lenders’ utility \( w^{H}, \) solving (60), together with \( w^{L} \) derived from the incentive constraint (59) satisfies (52). That is to say, the relevant case for the incentive constraint (34) is indeed case 2, that is (53).

Participation constraint binds only for \( \lambda^{H} \)

Consider now the case where \( C_{i}^{1} = 0 \) and the relevant participation constraint is slack. Because \((1 - \lambda^{L}) \theta > (1 - \lambda^{H}) \theta, \) it must be that the participation constraint for a \( \lambda^{L} \)-borrower is slack. At the same time, from lemma 17, we have: the limited commitment constraints bind for both types, \( C_{i}^{1} = (1 - \lambda^{i}) \theta, \forall i; \) the participation constraint binds for the \( \lambda^{H} \) type \( \alpha C_{i}^{1} + p(1 - \lambda^{H}) \theta = \omega, \) and the incentive constraint binds, \( u(X_{i}^{L}) = (1 - p) u(X_{i}^{H}) + pu(0) + \frac{\gamma}{(1 - q)}. \)
From the resource constraint at $t = 2$ we then have:

$$qw^H + (1 - q)u^{-1} \left( (1 - p) \ w^H + pu(0) + \frac{\gamma}{(1 - q)} \right) =$$

$$(1 - q) \omega + q \frac{p(1 - \lambda^H) \theta}{\alpha} + p\theta \left[ q \lambda^H + (1 - q) \lambda^L \right] =$$

$$(1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} \right) + (1 - q) \lambda^L \right]$$ \hspace{1cm} (61)

which gives the lender’s equilibrium utility if matched with a $\lambda^H$-borrower: $w^H$.

This equilibrium then exists if an only if

1. equation (61) has a solution, that is if there exists $w^H \in [0, \infty)$ such that:

$$qu^{-1}(w^H) = (1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} \right) + (1 - q) \lambda^L \right] -$$

$$(1 - q)u^{-1} \left( (1 - p) \ w^H + pu(0) + \frac{\gamma}{(1 - q)} \right)$$

If $u(0) = 0$ and $u$ is bounded then there exists such $w^H \in [0, \infty)$ and it is unique if and only if:

$$(1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} \right) + (1 - q) \lambda^L \right] > (1 - q)u^{-1} \left( \frac{\gamma}{(1 - q)} \right)$$

2. $(1 - \lambda^L) \theta > \frac{\alpha \omega}{p} > (1 - \lambda^H) \theta$ otherwise $C^L_1 = 0$ and $C^H_1 > 0$ could not be part of a solution to $(P2^F1)$.

3. lenders’ utility $w^H$, solving (61), together with $w^L$ derived from the incentive constraint (59) satisfies (52). That is to say, the relevant case for the incentive constraint (34) is indeed case 2, that is (53).

**Participation constraint slack $\forall i$**

Consider finally the case where $C^L_1 = 0$, $\forall i$ and the participation constraint is slack for both types: $C_{2h}^L = (1 - \lambda^L) \theta \forall i$, $C_1^H = C^L_1 = 0$ and still $u(X_2^L) = (1 - p) u (X_2^H) + pu(0) + \frac{\gamma}{(1 - q)}$.

From the resource constraint at $t = 2$ we then have:

$$qu^{-1}(w^H) + (1 - q)u^{-1} \left( (1 - p) \ w^H + pu(0) + \frac{\gamma}{(1 - q)} \right) =$$

$$\omega + p\theta[q \lambda^H + (1 - q) \lambda^L]$$ \hspace{1cm} (62)

which gives the lender’s equilibrium utility if matched with a $\lambda^H$-borrower: $w^H$.

This equilibrium then exists if an only if
1. equation (62) has a solution, that is if there exists $w^H \in [0, \infty)$ such that:

$$qu^{-1}(w^H) = \omega + p\theta[q\lambda^H + (1 - q)\lambda^L] - (1 - q)u^{-1}\left((1 - p)w^H + pu(0) + \frac{\gamma}{(1 - q)}\right)$$

If $u(0) = 0$ then there exists such $w^H \in [0, \infty)$ and it is unique if and only if:

$$\omega + p\theta[q\lambda^H + (1 - q)\lambda^L] > (1 - q)u^{-1}\left(\frac{\gamma}{(1 - q)}\right)$$

2. $(1 - \lambda^H) \theta > \frac{\omega}{p}$, otherwise $C_1^H = C_1^L = 0$ would not be part of a solution to $(P2^{FI})$.

3. lenders' utility $w^H$, solving (62), together with $w^L$ derived from the incentive constraint (59) satisfies (52). That is to say, the relevant case for the incentive constraint (34) is indeed case 2, that is (53).

### 6.2 Contract without screening, pooling over $\lambda^H$

So if screening is so costly that a $L$ does not want to do it in order to get $V^{FI}$ then any other strategy, in order to be part of an equilibrium in pure strategies, must prescribe no screening.

With no screening there are only two possible contracts in pure strategies that the CCP can offer: a pooling contract over $\lambda^H$-borrowers, where only the LC of the $\lambda^H$-borrower is satisfied, or a pooling contract over $\lambda^L$-borrowers, where also the LC of the $\lambda^L$-borrower is satisfied.

The first one, whose payoff we denote $V^{CCP,\lambda H}$, solves:

$$(P^H) \quad V^{CCP,\lambda H} = \max_{X_2^H, X_2^L} \quad qu(X_2^H) + (1 - q)u(X_2^L) \quad \text{s.t.} \quad \begin{align*}
\alpha C_1 + pC_{2h} + (1 - p)C_{2l} &\geq \omega \quad (64) \\
C_{2h} &\geq (1 - \lambda^H) \theta \quad (65) \\
C_1 &\leq \omega \quad (66) \\
qX_2^H + (1 - q)X_2^L + qpC_{2h} + (1 - q)p \max(C_{2h}, (1 - \lambda^L) \theta) + +q(1 - p)C_{2l} &\leq \omega - C_1 + p\theta \quad (67)
\end{align*}$$

**Lemma 18** A solution to $(P^H)$ is such that: i) the resource constraint at $t = 1$, (66), is always slack; ii) the resource constraint at $t = 2$, (67), always binds; iii) $X_2^H = X_2^L$; iv) with $\alpha > 1$ then $C_{2,l} = 0$.

**Proof.** i) Suppose not. Then $C_1 = \omega$ and the participation constraint (64) must be slack if $C_{2,l}$ satisfies the limited commitment constraint (65). Consider then the allocation defined by $C_1 = C_1 - \varepsilon$ for $\varepsilon > 0$ arbitrarily small, and $X_2^i = X_2^i + \alpha \varepsilon$ for $i = L, H$. This allocation is still in the constraint set of problem $(P^H)$ and yields higher value of the objective.
ii) Suppose not. Consider then the allocation defined by $\hat{X}_i^2 = X_i^2 + \varepsilon$ for $i = L, H$, for $\varepsilon > 0$ arbitrarily small so that the resource constraint at $t = 2$, (67), is still satisfied. This allocation is still in the constraint set of problem ($P^H$) and yields higher value of the objective.

iii) Suppose not. Consider the case $X_i^H > \hat{X}_i^L$ and the alternative allocation defined by $\hat{X}_i^2 = X_i^2 - \varepsilon$ and $\hat{X}_i^L = X_i^L + \frac{h}{1-q} \varepsilon$ for $\varepsilon > 0$ arbitrarily small so that it still is the case that $\hat{X}_i^H > \hat{X}_i^L$. This allocation is still in the constraint set of problem ($P^H$) and the objective, evaluated at $\{\hat{X}_i^2\}_{t=2}^{L, H}$, is $u(\hat{X}_i^L) + q(u(\hat{X}_i^H) - u(\hat{X}_i^L))$. The change in the value of the objective with respect to the original allocation, $\Delta u$, is then: $\Delta u = \varepsilon [u'(\hat{X}_i^L) \frac{q}{1-q} + q(-u'(\hat{X}_i^H) - u'(\hat{X}_i^L) \frac{q}{1-q})]$ which is simply $\Delta u = \varepsilon q[u'(\hat{X}_i^L) - u'(\hat{X}_i^H)]$. Because $u$ is strictly concave and $\hat{X}_i^2 > \hat{X}_i^L$, then $\Delta u > 0$.

iv) Suppose not. Then $C_{2, L} > 0$: consider the allocation defined by $\hat{C}_{2, L} = C_{2, L} - \varepsilon$, $\hat{C}_1 = C_1 + \frac{\varepsilon}{\alpha}$ and $\hat{X}_i^2 = X_i^2 + \varepsilon(1 - \frac{1}{\alpha})$, for $i = L, H$. Because $\alpha > 1$ then $\hat{X}_i^2 > X_i^1$, $i = L, H$. Therefore this allocation is still in the constraint set to problem ($P^H$) and yields higher value of the objective.

Thus the CCP problem ($P^H$) is simply:

$$V^{CCP, LH} = \max \ u \left[ - \omega - C_1 + p \theta - q p C_{2h} - (1 - q) p \max (C_{2h}, (1 - \lambda L) \theta) \right]$$

s.t. $\alpha C_1 + p C_{2h} + (1 - p) C_{2L} \geq \alpha \omega$

$C_{2h} \geq (1 - \lambda^H) \theta$

So either both constraints bind, or the LC binds and the PC is slack. Notice that with a CCP, as opposed to the bilateral problem, it never happens that the LC is slack and PC binds in order to bring $\omega$ goods over to $t = 2$ for insurance purposes, as the bilateral clearing requires when $\omega$ is scarce, which is the collateral and the only means to provide insurance to the lender against the $\theta$ shock. Also, as in the other problems, $\alpha > 1$ implies that the solution to the CCP problem is such that $C_{2L} = 0$.

1. LC and PC bind: $C_{2h} = (1 - \lambda^H) \theta$ and $\alpha C_1 + p (1 - \lambda^H) \theta = \alpha \omega$. So the value of the objective is:

$$V^{CCP, LH} = u \left( \frac{(1 - \lambda^H) p \theta}{\alpha} + p \theta - q p \left(1 - \lambda^H\right) \theta - (1 - q) p \left(1 - \lambda^L\right) \theta \right)$$

$$= u \left( \frac{(1 - \lambda^H) p \theta}{\alpha} + p \theta \left[q \lambda^L + (1 - q) \lambda^H\right]\right)$$

(68)

this is an equilibrium if and only if $p (1 - \lambda^H) \theta \leq \alpha \omega$.

2. LC binds and PC is slack: $C_{2h} = (1 - \lambda^H) \theta$, $C_1 = 0$ and $p (1 - \lambda^H) \theta > \alpha \omega$. So the value of the objective is:

$$V^{CCP, LH} = u \left( \omega + p \theta - q p \left(1 - \lambda^H\right) \theta - (1 - q) p \left(1 - \lambda^L\right) \theta \right)$$

51
\[ u (\omega + p \theta [q \lambda^H + (1 - q) \lambda^L]) \]

this is an equilibrium if and only if \( p (1 - \lambda^H) \theta > \alpha \omega \).

6.3 Contract without screening, pooling over \( \lambda^L \)

This problem, whose payoff we denote \( V^{CCP,\lambda L} \) is the same as described in section 4.

\[
V^{CCP,\lambda L} = \max \ u (X_2) \\
\text{s.t.} \quad \alpha C_1 + p C_{2h} + (1 - p) C_{2l} \geq \alpha \omega \\
C_{2h} \geq (1 - \lambda^L) \theta \\
C_1 \leq \omega \\
X_2 + p C_{2h} + (1 - p) C_{2l} \leq \omega - C_1 + p \theta
\]

where \( X^H_2 = X^L_2 \) has been already substituted out, together with \( C_{2l} = 0 \), which, as in the previous problem, always holds as long as \( \alpha > 1 \).

Similarly to the previous case, either both constraints bind, or the LC binds and the PC is slack. It is never optimal for the CCP to have the limited commitment constraint slack and the participation constraint bind, which, in the lender’s problem with bilateral clearing, was necessary and in some cases optimal in order to insure the lender against the borrower’s output shock.

1. LC and PC binds: \( C_{2h} = (1 - \lambda^L) \theta \) and \( \alpha C_1 + p (1 - \lambda^L) \theta = \alpha \omega \). So the value of the objective is:

\[
V^{CCP,\lambda L} = u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} + p \theta - (1 - \lambda^L) \theta \right) \\
= u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} + p \theta \lambda^L \right)
\]

this is an equilibrium if and only if \( p (1 - \lambda^L) \theta \leq \alpha \omega \).

2. LC binds and PC is slack: \( C_{2h} = (1 - \lambda^L) \theta \), \( C_1 = 0 \) and \( p (1 - \lambda^L) \theta > \alpha \omega \). So the value of the objective is:

\[
V^{CCP,\lambda L} = u \left( \omega + p \theta \lambda^L \right)
\]

this is an equilibrium if and only if \( p (1 - \lambda^L) \theta > \alpha \omega \).

6.4 CCP equilibrium

Since the focus of the paper is to characterize the trade-off between central and bilateral clearing with respect to the loss in lenders’ incentives to acquire information about their borrowers, then this section characterizes equilibria where clearing via a CCP results in lenders offering contracts without information
acquisition. That is to say pooling contracts. In the first subsection we character-
ize equilibria with a pooling contract over $\lambda^L$ when clearing through a CCP,
while the second subsection studies the case of equilibria with a pooling contract
over $\lambda^H$. In either case, economies where these equilibria arise are charac-
terized by a loss of information about the borrowers’ types.

6.4.1 Equilibrium with pooling over $\lambda^L$

When looking at equilibria in pure strategies with no screening, the CCP offers
the contract that yields higher payoff to lenders. The contract that collapses all
borrowers over $\lambda^L$ types is offered if and only if $V_{\text{CCP},\lambda^L} \geq V_{\text{CCP},\lambda^H}$.

If $p(1 - \lambda^L) \theta \leq \alpha \omega$ then it is also true that $p(1 - \lambda^H) \theta \leq \alpha \omega$, thus
$V_{\text{CCP},\lambda^L} \geq V_{\text{CCP},\lambda^H}$ if and only if:

$$u \left( \frac{(1 - \lambda^L) p \theta}{\alpha} + p \theta \lambda^L \right) \geq u \left( \frac{(1 - \lambda^H) p \theta}{\alpha} + p \theta [q \lambda^H + (1 - q) \lambda^L] \right)$$

$$\frac{(\lambda^H - \lambda^L) p \theta}{\alpha} \geq p \theta [q \lambda^H + (1 - q) \lambda^H - \lambda^L]$$

$1 \geq \alpha q$

If $p(1 - \lambda^H) \theta \leq \alpha \omega < p(1 - \lambda^L) \theta$ then $V_{\text{CCP},\lambda^L} \geq V_{\text{CCP},\lambda^H}$ if and only if:

$$u (\omega + p \theta \lambda^L) \geq u \left( \frac{(1 - \lambda^H) p \theta}{\alpha} + p \theta [q \lambda^H + (1 - q) \lambda^L] \right)$$

$$\omega \geq \frac{(1 - \lambda^H) p \theta}{\alpha} + p \theta q (\lambda^H - \lambda^L)$$

In words, for the CCP contract that treats every borrower as a bad borrower
(i.e. the pooling contract over $\lambda^L$) to be preferred by lenders, the amount of
collateral available in the economy must be large enough to offset the aggregate
cost, in terms of resources, of facing a more severe commitment problem for those
borrowers who are $\lambda^H$ but are treated as $\lambda^L$ and thus receive more $C_{2h}$ than
they otherwise would. For every such borrower, the cost in terms of additional
resources is $\theta (\lambda^H - \lambda^L)$ in the state of the world where the output realization
is $\theta$, and zero otherwise. Because in this economy the law of large numbers
hold, then the aggregate additional resources necessary to offset this cost are
$p \theta q (\lambda^H - \lambda^L)$.

In the contract where only the LC constraint for $\lambda^H$-borrowers is considered
the cost is instead forgone $C_1$ on $\lambda^L$-borrowers: in fact, with respect to a $\lambda^L$-
borrower, a $\lambda^H$-borrower is paid less $C_{2h}^H$ which requires a higher $C_1^H$ for the
PC to be satisfied. Thus in such a contract the cost is the additional resources
at $t = 1$ paid to the measure $(1 - q)$ of $\lambda^L$-borrowers who would be paid
$C_1^L$ in a full information allocation but are paid $C_1^H$ for $C_{2h}^H > C_1^L$. And because $\lambda^L$-
borrowers also consume $(1 - \lambda^L) \theta > C_{2h}^H$, then overall the expected utility of
these borrowers (equal to the aggregate resources transferred to them) exceeds that of \( \lambda^H \)-borrowers.

If \( \alpha \omega < p \left( 1 - \lambda^L \right) \theta \) then \( V^{CCP,\lambda L} \geq V^{CCP,\lambda H} \) if and only if:

\[
u \left( \omega + p\theta \lambda^L \right) \geq u \left( \omega + p\theta \left[ q\lambda^H + (1 - q) \lambda^L \right] \right)\]

but this is never possible. So when the PC is slack in both problems the CCP always offers the contract that satisfies the \( \lambda^H \) borrower LC constraint. In fact, when the PC is slack, the only way to induce the borrower to trade is to pay him \( C_{2h} \), because \( C_1 = 0 \). Therefore it is cheaper to pay sufficient \( C_{2h} \) to satisfy \( V^{CCP,\lambda H} \) because with \( C_1 = 0 \), the CCP would not save anything by trying to pay less \( C_1 \) to \( \lambda^H \)-borrowers who are treated as \( \lambda^L \), which happens in the contract that delivers \( V^{CCP,\lambda L} \).

**Lemma 19** \( V^{CCP,\lambda L} \geq V^{CCP,\lambda H} \) if and only if \( 1 \geq \alpha q \) when \( p \left( 1 - \lambda^L \right) \theta \leq \alpha \omega \) and if and only if \( \omega \geq \frac{(1 - \lambda^H)p\theta}{\alpha} + \lambda^H \lambda^L \) when \( p \left( 1 - \lambda^H \right) \theta \leq \alpha \omega < p \left( 1 - \lambda^L \right) \theta \). Otherwise, when \( \alpha \omega < p \left( 1 - \lambda^H \right) \theta \), \( V^{CCP,\lambda L} < V^{CCP,\lambda H} \) always.

**Corollary 20** Assume \( u(0) = 0 \), \( p \left( 1 - \lambda^L \right) \theta \leq \alpha \omega \) and \( 1 \geq \alpha q \) and define:

\[
\lambda^H_1 = \frac{\alpha \left[ qu^{-1} \left( V^{CCP,\lambda L} + \gamma \frac{(1-q)}{q} \right) + (1-q)u^{-1} \left( V^{CCP,\lambda L} - \gamma \right) \right]}{q\theta(\alpha - 1)} - \frac{1}{q(\alpha - 1)} - \frac{(1-q)\lambda^L}{q} \tag{70}
\]

\[
\lambda^H_2 = \frac{\alpha \left[ qu^{-1} \left( V^{CCP,\lambda L} - \gamma \right) + (1-q)u^{-1} \left( \frac{(1-p)(1-q)\lambda^H}{(1-q)(1-p)(1-q)} \right) \right]}{q\theta(\alpha - 1)} - \frac{1}{q(\alpha - 1)} - \frac{(1-q)\lambda^L}{q} \tag{71}
\]

then \( V^{CCP,\lambda L} \geq V^{FI} \) if and only if \( \lambda^H < \min(\lambda^H_1, \lambda^H_2) \), where \( V^{CCP,\lambda L} = u \left( \frac{(1-\lambda^L)p\theta}{\alpha} + p\theta \lambda^L \right) \).

**Proof.** When \( p \left( 1 - \lambda^L \right) \theta \leq \alpha \omega \) then in the CCP contract that separates borrowers’ types, both participation constraints bind. Suppose \( |40| \) holds so that the incentive constraint in problem \( (P_0^{FI}) \) is \( |42| \), which is binding at a solution. Therefore \( V^{FI} = qw^H + (1-q)w^L - \gamma = w^L \). One of the necessary and sufficient conditions for such an equilibrium to exist is \( \gamma < qu \left( \frac{\omega}{\alpha} \left[ \frac{1}{\alpha} + \left( q\lambda^H + (1-q) \lambda^L \right) \right] \right) \).

Also, the payoff from the pooling CCP contract over \( \lambda^L \) is \( V^{CCP,\lambda L} = u \left( \frac{(1-\lambda^L)p\theta}{\alpha} + p\theta \lambda^L \right) \).

Therefore \( V^{CCP,\lambda L} > V^{FI} \) if and only if \( u \left( \frac{(1-\lambda^L)p\theta}{\alpha} + p\theta \lambda^L \right) > q \left( w^L + \frac{\gamma}{q} \right) + (1-q)w^L - \gamma \) that is simply \( u \left( \frac{(1-\lambda^L)p\theta}{\alpha} + p\theta \lambda^L \right) > w^L \). Recall that \( w^L \) is defined by

\[
qu^{-1} \left( w^L + \frac{\gamma}{q} \right) + (1-q)u^{-1} (w^L) = p\theta \left[ \frac{1}{\alpha} + \left( q\lambda^H + (1-q) \lambda^L \right) \right] \]

54
where the left hand side is monotonically increasing and the right hand side is constant. Then we can then define \( \hat{w}_1 = u \left( \frac{(1-\lambda^L)p\alpha + p\theta\lambda^L}{1-p(1-q)} \right) \) and \( V_{CCP,\lambda^L} > V^{FI} \) if and only if

\[
qu^{-1} \left( \hat{w}_1 + \frac{\gamma}{q} \right) + (1-q) u^{-1} (\hat{w}_1) > p\theta \left[ \frac{1}{1-q} + (q\lambda^H + (1-q)\lambda^L) \left( 1 - \frac{1}{\alpha} \right) \right]
\]

which is equivalent to \( \lambda^H < \lambda^H_1 \) with \( \lambda^H_1 \) defined in (70).

Consider now the case when (52) holds so that the incentive constraint in problem \((P0)^{FI}\) is (53), which is binding at a solution and can be rewritten as

\[
w^L = (1-p)w^H + \frac{(1-q)}{1-q}
\]

One of the necessary and sufficient conditions for this equilibrium to exists is:

\[
\frac{\gamma}{1-q} < u \left( p\frac{\theta}{1-q} \left[ q \left( \lambda^H + \frac{(1-\lambda^H)}{\alpha} \right) + (1-q) \left( \lambda^L + \frac{(1-\lambda^L)}{\alpha} \right) \right] \right)
\]

Then \( V^{FI} = w^H [1-p(1-q)] \), so \( V_{CCP,\lambda^L} > V^{FI} \) if and only if

\[
u \left( \frac{1-\lambda^L}{\alpha} p\theta + p\theta\lambda^L \right) > w^H [1-p(1-q)]
\]

Recall that \( w^H \) is defined by

\[
qu^{-1} (w^H) + (1-q) u^{-1} (1-p) w^H + \frac{\gamma}{1-q} = p\theta \left[ \frac{1}{1-q} + (q\lambda^H + (1-q)\lambda^L) \left( 1 - \frac{1}{\alpha} \right) \right]
\]

where the left hand side is monotonically increasing and the right hand side is constant. Then we can then define \( \hat{w}_2 = \frac{u \left( \frac{(1-\lambda^L)p\alpha + p\theta\lambda^L}{1-p(1-q)} \right)}{1-q} \) and \( V_{CCP,\lambda^L} > V^{FI} \) if and only if

\[
qu^{-1} (\hat{w}_2) + (1-q) u^{-1} (1-p) \hat{w}_2 + \frac{\gamma}{1-q} > p\theta \left[ \frac{1}{1-q} + (q\lambda^H + (1-q)\lambda^L) \left( 1 - \frac{1}{\alpha} \right) \right]
\]

which is equivalent to \( \lambda^H < \lambda^H_2 \) with \( \lambda^H_2 \) defined in (71).

If \( u(0) > 0 \) the results in corollary 20 go through redefining \( \lambda^H_1 \) and \( \lambda^H_2 \) accordingly.

Notice that in the case where the incentive compatibility is as in case 2 then the solution to the CCP problem \((P1)^{FI}\) must be such that \( w^L < w^H \) otherwise the relevant incentive compatibility would be as in case 1: in fact \( w^L < (q + (1-q)(1-p))w^H + (1-q)pu(0) \), when \( u(0) = 0 \), implies \( w^L < w^H \).

In general, for the CCP pooling contract over \( \lambda^L \) to be preferred by lenders to the full information contract it must be that the cost of monitoring is sufficiently high and the value of a \( \lambda^H \) borrower to the coalition of lenders is

\[28\] Where \( u(0) = 0 \) has been substituted out.
not too high. Recall that this is an economy relatively rich of collateral, since 
\( \alpha \omega \geq p (1 - \lambda^L) \theta \) and where there are relatively few good borrowers, since 
1 \( \geq \alpha q \). In the CCP problem, collateral is never used for insurance purposes, 
since insurance is achieved via risk pooling, but every unit of consumption good 
that is stored to \( t = 2 \) is similar to a wealth effect because the CCP can pay 
higher \( w^H, w^L \) regardless of whether the output realization is high or low. Thus, 
the benefit of saving resources by monitoring borrower to find out the good ones 
must be traded off with the cost of monitoring and the shadow value of the incentive 
constraint for lenders. Clearly, the trade off with the cost of monitoring 
boils down to a lower bound on \( \gamma \). The trade of with the cost of satisfying 
lenders’ incentive constraint for the CCP is that some insurance across bor-
rowers must be forgone: thus a lender matched with a good borrower must be 
be sufficiently rewarded: that is to say \( w^H \) must be sufficiently larger than \( w^L \).

When the incentive constraint is as in case 1 then it must be that \( (1 - q) p \) is 
sufficiently large, so that a lender does not want to get caught having reported 
a \( \lambda^H \) type when the borrower turns out to be a \( \lambda^L \) type. Therefore the only 
way that the CCP could induce lenders to monitor borrowers rather than just 
reporting a \( \lambda^H \) type is to pay a high \( w^H \). If there are enough bad borrowers 
\( ((1 - q) p \) is sufficiently large) then this is still consistent with the incentive 
constraint in case 1: otherwise a high \( w^H \) would imply an incentive constraint 
as in case 2. Thus, given a relatively large fraction of \( \lambda^L \) borrowers (so \( (1 - q) p \) 
is sufficiently large) it is necessary that the monitoring cost \( \gamma \) is not too high for 
the CCP relative to the \( w^H \) to implement the full information contract. If these 
conditions are not satisfied then the CCP pooling contract over \( \lambda^L \) is preferred.

When the incentive constraint is as in case 2 instead, there are relatively 
many good borrowers: thus paying a high \( w^H \) over \( w^L \) drains a lot of resources 
from the CCP. Thus, for the CCP to implement the full information contract \( w^H \) 
cannot be too large. As a result, the full information contract is implementable 
only for a smaller set of monitoring costs \( \gamma \).

**Corollary 21** Assume \( u(0) = 0, (1 - \lambda^H) \frac{\rho\theta}{\alpha} + p\theta (\lambda^H - \lambda^L) \leq \omega \) < \( (1 - \lambda^L) \frac{\rho\theta}{\alpha} \) and define:

\[
\lambda^H_1 = \alpha \left[ qu^{-1}(V_{CCP,\lambda^L} + \gamma \frac{(1-q)}{q}) + (1-q)u^{-1}(V_{CCP,\lambda^L} - \gamma) - (1-q)\omega \right] \div \frac{q\theta(\alpha - 1)}{\omega} \leq \frac{1}{\alpha - 1} - \frac{\alpha(1-q)}{\alpha(\alpha - 1)} \lambda^L
\]

\[
\lambda^H_2 = \alpha \left[ qu^{-1}(V_{CCP,\lambda^L} - \gamma) + (1-q)u^{-1}(1-q)\frac{(1-q)\gamma V_{CCP,\lambda^L} - \gamma}{q\theta(1-q)} - (1-q)\omega \right] \div \frac{q\theta(\alpha - 1)}{\omega} \leq \frac{1}{\alpha - 1} - \frac{\alpha(1-q)}{\alpha(\alpha - 1)} \lambda^L
\]

then \( V_{CCP,\lambda^L} \geq V^{FI} \) if and only if \( \lambda^H < \min(\lambda^H_1, \lambda^H_2) \), where \( V_{CCP,\lambda^L} = u(\omega + p\theta\lambda^L) \).

**Proof.** When \( (1 - \lambda^H) \frac{\rho\theta}{\alpha} + p\theta (\lambda^H - \lambda^L) \leq \omega \) < \( (1 - \lambda^L) \frac{\rho\theta}{\alpha} \) then in the CCP 
contract that separates borrowers’ types, only the participation constraint for 
the \( \lambda^H \) type binds. Suppose first that \( (40) \) holds so that the incentive constraint 
in problem \( (P\theta^{FI}) \) is \( (42) \), which is binding at a solution. Therefore \( V^{FI} = w^L \).
One necessary and sufficient condition of this equilibrium to exist is:

\[
\frac{\gamma}{q} < u \left( \frac{1 - q}{q} \omega + p\theta \left[ \lambda^H + \frac{(1 - \lambda^H)}{\alpha} + \frac{(1 - q)}{q} \lambda^L \right] \right)
\]

In this equilibrium \( w^L \) solves (50), which, together with the binding incentive constraint (42) determines \( w^H \). Because \( u \) is strictly increasing then \( V^{CCP,\lambda L} \geq V^{FI} \) if and only if:

\[
qu^{-1} \left( V^{CCP,\lambda L} + \frac{\gamma}{q} \right) + (1 - q) u^{-1} \left( V^{CCP,\lambda L} \right) > (1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} + (1 - q) \lambda^L \right) \right] \]

(74)

where \( V^{CCP,\lambda L} = u(\omega + p\theta \lambda^L) \), because, in this case, \( \omega < (1 - \lambda^L) \frac{p\theta}{\alpha} \). Because \( V^{CCP,\lambda L} \) depends only on \( \lambda^L \) but not on \( \lambda^H \), then (74) can be rewritten as an upper bound on \( \lambda^H \):

\[
\alpha \left[ qu^{-1} \left( V^{CCP,\lambda L} + \frac{\gamma}{q} \right) + (1 - q) u^{-1} \left( V^{CCP,\lambda L} \right) - (1 - q) \omega \right] + q(\alpha - 1)p\theta \left[ -\frac{1}{\alpha - 1} - \frac{\alpha(1 - q)}{q(\alpha - 1)} \lambda^L \right] > \lambda^H
\]

(75)

Let \( \lambda^H \) denote the left hand side of (75).

Consider now the case where (52) holds so that the incentive constraint in problem \( (P0^{FI}) \) is (53), which is binding at a solution. Therefore \( V^{FI} = w^H [1 - p(1 - q)] \). One necessary and sufficient condition for this equilibrium to exist is:

\[
(1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} \right) + (1 - q) \lambda^L \right] > (1 - q) u^{-1} \left( \frac{\gamma}{(1 - q)} \right)
\]

In this equilibrium \( w^H \) solves (61), which, together with the binding incentive constraint (53) determines \( w^L \). Because \( u \) is strictly increasing then \( V^{CCP,\lambda L} \geq V^{FI} \) if and only if \( \frac{V^{CCP,\lambda L}}{1 - p(1 - q)} > w^H \), where \( V^{CCP,\lambda L} = u(\omega + p\theta \lambda^L) \), because \( \omega < (1 - \lambda^L) \frac{p\theta}{\alpha} \). Using (61) and the fact that \( u \) is strictly increasing, then \( V^{CCP,\lambda L} \geq V^{FI} \) if and only if:

\[
qu^{-1} \left( \frac{V^{CCP,\lambda L}}{1 - p(1 - q)} \right) + (1 - q) u^{-1} \left( \frac{V^{CCP,\lambda L}}{1 - p(1 - q)} + \frac{\gamma}{(1 - q)} \right) > (1 - q) \omega + p\theta \left[ q \left( \lambda^H + \frac{(1 - \lambda^H)}{\alpha} \right) + (1 - q) \lambda^L \right]
\]

(76)

As in the case of (74), and assuming \( u(0) = 0 \), this can be rewritten as an upper bound on \( \lambda^H \):

\[
\alpha \left[ qu^{-1} \left( \frac{V^{CCP,\lambda L}}{1 - p(1 - q)} \right) + (1 - q) u^{-1} \left( \frac{V^{CCP,\lambda L}}{1 - p(1 - q)} + \frac{\gamma}{(1 - q)} \right) - (1 - q) \omega \right] + q(\alpha - 1)p\theta
\]

57
\[-\frac{1}{\alpha - 1} - \frac{\alpha(1 - q)}{q(\alpha - 1)} \lambda^L > \lambda^H \]  

(77)

Let \( \lambda_2^H \) denote the left hand side of \( (77) \). ■

If \( u(0) > 0 \) the results in corollary 21 go through redefining \( \lambda_1^H \) and \( \lambda_2^H \) accordingly.

6.4.2 Equilibrium with pooling over \( \lambda^H \)

There are two cases of equilibria without screening and a pooling contract over \( \lambda^H \): if \( \omega \geq \frac{\theta(1 - \lambda^L)}{\alpha} \) then both the limited commitment and the participation constraints, (65) and (64), bind in problem \((P^H)\). This results in a payoff to lenders given by (68). In this case the CCP contract that induces screening is such that either both participation constraints bind in problem \((P^{0F})\) or only the participation constraint for \( \lambda^H \) types. Also, the pooling contract over \( \lambda^H \) is preferred to the pooling contract over \( \lambda^L \) if and only if:

a. either \( \alpha q > 1 \) when also \( \omega \geq \frac{\theta(1 - \lambda^L)}{\alpha} \)

b. or \( \omega < \frac{\theta(1 - \lambda^L)}{\alpha} \)

If instead \( \omega < \frac{\theta(1 - \lambda^H)}{\alpha} \) then the limited commitment constraint binds and the participation constraint is slack in problem \((P^H)\). This results in a payoff to lenders given by (69). In this case both participation constraints are slack the CCP contract that induces screening. Also, the pooling contract over \( \lambda^H \) is always preferred to the pooling contract over \( \lambda^L \) in this case.

**Case a.** \( \omega \geq \frac{\theta(1 - \lambda^L)}{\alpha} \) and \( \alpha q > 1 \) Pooling over \( \lambda^H \) is preferred to pooling over \( \lambda^L \). With respect to the CCP contract with screening in problem \((P^H)\), in this case we also have \( \omega \geq \frac{\theta(1 - \lambda^H)}{\alpha} \): thus we consider two sub-cases: the participation constraints for both types bind in \((P^{0F})\) and only the participation constraint for \( \lambda^H \) binds. Also, there can be two cases for the incentive constraint (34) either (40) holds so that the relevant incentive constraint is (42); or (52) holds so that the relevant incentive constraint is (53).

a.1 Suppose first that (40) holds so that the relevant incentive constraint is (42) and the participation constraints for both types bind in \((P^{0F})\). In this case \( V^{FI} = w^L \) and \( w^L \) is defined by (49). Also, in this case

\[ V_{CCP,\lambda^H} = u \left( \frac{1 - \lambda^H}{\alpha} \right) + p \theta \left[ q \lambda^H + (1 - q) \lambda^L \right] \]

as defined in (68). Thus \( V_{CCP,\lambda^H} > V^{FI} \) if and only if \( V_{CCP,\lambda^H} > w^L \). Using (49), this is equivalent to:

\[ qu^{-1} \left( V_{CCP,\lambda^H} + \frac{2}{q} \right) + (1 - q) u^{-1} \left( V_{CCP,\lambda^H} \right) > p \theta \left[ \frac{1}{\alpha} + (q \lambda^H + (1 - q) \lambda^L) \left( 1 - \frac{1}{\alpha} \right) \right] \]

(78)

58
which, in turn, can be rewritten as a lower bound on $\gamma$ as follows. Let $\gamma$ denote the value of $\gamma \in (0,1)$ such that (78) holds at equality:

$$\gamma = qu \left( \frac{p\theta}{q} \left[ \frac{1}{\alpha} + (q\lambda^H + (1-q)\lambda^L) \left(1 - \frac{1}{\alpha}\right) \right] - (1-q)u^{-1}(V_{CCP,\lambda^H}) \right) - qV_{CCP,\lambda^H}$$

(79)

Therefore, in this case, the CCP equilibrium is pooling over $\lambda^H$ if and only if $\gamma \geq \tilde{\gamma}$.  

a.2 Suppose now that (52) holds so that the relevant incentive constraint is (53) and the participation constraints for both types bind in $(P0^F)$. In this case $V_{FI} = w^H (1-p(1-q))$ and $w^H$ is defined by (60). Also, in this case $V_{CCP,\lambda^H} = u \left( \frac{(1-\lambda^H)p\theta}{\alpha} + p\theta \left[ q\lambda^H + (1-q)\lambda^L \right] \right)$ as defined in (68). Thus $V_{CCP,\lambda^H} > V_{FI}$ if and only if $\frac{V_{CCP,\lambda^H}}{1-p(1-q)} > w^H$. Using (60) and the fact that $u$ is strictly increasing, this is equivalent to:

$$qu^{-1}(\frac{V_{CCP,\lambda^H}}{1-p(1-q)}) + (1-q)u^{-1}(\frac{(1-p)V_{CCP,\lambda^H}}{1-p(1-q)} + \frac{\gamma}{1-q}) > p\theta \left[ q \left( \lambda^H + \frac{(1-\lambda^H)}{\alpha} \right) + (1-q) \left( \lambda^L + \frac{(1-\lambda^L)}{\alpha} \right) \right]$$

(80)

which, in turn, can be rewritten as a lower bound on $\gamma$ as follows. Let $\gamma$ denote the value of $\gamma \in (0,1)$ such that (80) holds at equality:

$$\gamma = (1-q)u \left( \frac{p\theta}{1-q} \left[ q \left( \lambda^H + \frac{(1-\lambda^H)}{\alpha} \right) + (1-q) \left( \lambda^L + \frac{(1-\lambda^L)}{\alpha} \right) \right] - \frac{q}{1-q} u^{-1}(\frac{V_{CCP,\lambda^H}}{1-p(1-q)}) \right) - \frac{(1-q)(1-p)V_{CCP,\lambda^H}}{1-p(1-q)}$$

Therefore, in this case, the CCP equilibrium is pooling over $\lambda^H$ if and only if $\gamma \geq \tilde{\gamma}$.  

7 Conclusions

We studied the efficient flow of information in an environment of Over-The-Counter trading with adverse selection. In an environment with limited commitment and private information, the clearing arrangement interacts with traders’ incentives to screen their counterparties. This has significant consequences for transparency and for collateral requirements. When clearing is bilateral, collateral plays two roles: it disciplines incentives and at the same time it provides

29Where $u(0) = 0$ has been substituted out.
insurance. When clearing is performed by a central counterparty (CCP), collateral is primarily used to discipline incentives. The CCP, by risk pooling, provides insurance against counterparty risk, at the cost of a loss in incentives to acquire information about the quality of traders’ counterparties. Such loss of information, if it occurs in equilibrium, is efficient. The desirability of central clearing, then, depends on the resolution of such a trade-off: market participants, absent any externality, always choose the efficient clearing arrangement.
Figure 2: CCP problem.
APPENDIX

Proof of Lemma 1

First, we show that collateral is always positive.

Claim 22 At a solution, constraint (3) is always slack.

Proof. Suppose not. Then if \( \{ c^i_1, c^i_2, c^i_{2,h}, c^i_{2,l} \}_{i=H,L} \) denotes a solution, it must be that \( c^i_{2,h} \geq (1 - \lambda_i) \theta \) and by assumption \( c^i_1 = \omega \). This implies that the participation constraint (2) is slack because \( c^i_1 > \omega \). Then we can construct an alternative allocation as follows: \( \hat{c}^i_1 = c^i_1 - \varepsilon \), \( \hat{c}^i_2 = c^i_{2,h} \), \( \hat{c}^i_{2,l} = c^i_{2,l} \).

All constraints are still satisfied, and expected utility of the lender increases, which is a contradiction.

Replacing \( x^i_{2,h} \) and \( x^i_{2,l} \), and ignoring constraint (3), we can rewrite the problem as

\[
\max \{ c^i_1, c^i_2, h, c^i_{2,l} \} \in \mathbb{R}^3_+ \quad \text{s.t.} \quad (\mu) \quad \alpha c^i_1 + pc^i_{2,h} + (1 - p)c^i_{2,l} \geq \alpha \omega \quad (82)
\]

\[
(\eta) \quad c^i_{2,h} \geq (1 - \lambda_i) \theta \quad (83)
\]

Let \( \mu \) and \( \eta \) be the multipliers associated with (82) and (83) respectively. The first order conditions are

\[
- p u'(\omega - c^i_1 + \theta - c^i_{2,h}) + p \mu + \eta = 0 \quad (84)
\]

\[
- (1 - p) u'(\omega - c^i_1 - c^i_{2,l}) + (1 - p) \mu \leq 0 \quad (85)
\]

with equality if \( c^i_{2,l} > 0 \) and

\[
- p u'(\omega - c^i_1 + \theta - c^i_{2,h}) - (1 - p) u'(\omega - c^i_1 - c^i_{2,l}) + \alpha \mu \leq 0 \quad (86)
\]

with equality if \( c^i_1 > 0 \). Together with the complementary slackness conditions

\[
\mu \{ \alpha c^i_1 + pc^i_{2,h} + (1 - p)c^i_{2,l} - \alpha \omega \} = 0 \quad (87)
\]

and

\[
\eta \{ c^i_{2,h} - (1 - \lambda_i) \theta \} = 0 \quad (88)
\]

these conditions characterize the solution to the problem.

Next we show that borrower’s consumption in the low state equals zero.

Claim 23 At a solution \( c^i_{2,l} = 0, \forall i \).
Proof. Suppose not. Then (85) must hold with equality and \( \mu = u'(\omega - c_1 - c_{2,l}) > 0 \) that replaced in (84) gives

\[
\eta = p\left[u'(\omega - c_1 + \theta - c_{2,h}^{i}) - u'(\omega - c_1 - c_{2,l})\right] \]

Because \( \eta \geq 0 \), it must be \( c_{2,h}^{i} \geq c_{2,l}^{i} + \theta > 1 - \lambda i \theta \), hence \( \eta = 0 \) and \( c_{2,h}^{i} = c_{2,l}^{i} + \theta \). Therefore, in (84) we obtain

\[
\mu = u'(\omega - c_1^{i} - c_{2,l}^{i}) \]

that replaced in (86) gives \( (\alpha - 1) \mu \leq 0 \) which contradicts \( \mu > 0 \) and \( \alpha > 1 \). 

Finally we prove that insurance is incomplete

Claim 24 If \( u \) is strictly increasing then a solution to problem (1) \( x_{2,h}^{i*} > x_{2,l}^{i*} \).

Proof. Suppose not. Then a solution to problem (1) is such that \( x_{2,h}^{i*} = x_{2,l}^{i*} \) since it is never the case that \( x_{2,h}^{i*} < x_{2,l}^{i*} \) because that would imply \( c_{2,h}^{i} > \theta \) and in turn that the limited commitment constraint is slack and that \( c_{2,l}^{i} < \omega \) (otherwise the participation constraint would also be slack, but then the lender could pay the borrower less so that it binds and attain higher utility). This can happen if and only if the limited commitment constraint is slack (\( \eta = 0 \)) and \( c_{2,l}^{i} = \theta \). Then it must be that \( c_{2,h}^{i*} < \omega \) (otherwise the participation constraint would also be slack) and we can construct an alternative allocation such that \( \hat{c}_{1}^{i*} = c_{1}^{i*} + \varepsilon, \hat{c}_{2,h}^{i*} = c_{2,h}^{i*} - \frac{\alpha \varepsilon}{1-p}, \hat{c}_{2,l}^{i*} = c_{2,l}^{i*} - \frac{\alpha \varepsilon}{1-p}, \hat{x}_{2,h}^{i*} = x_{2,h}^{i*} + \alpha \varepsilon, \hat{x}_{2,l}^{i*} = x_{2,l}^{i*} + \alpha \varepsilon \). This allocation is still feasible given that not all resources at \( t = 1 \) were used (\( c_{2,l}^{i*} < \omega \)) and it still satisfies the participation constraint and the limited commitment constraint for \( \varepsilon > 0 \) and arbitrarily small. Because \( u \) is strictly increasing and because \( \frac{\alpha}{p} > 1, \frac{\alpha}{1-p} > 1 \) , then \( \hat{x}_{2,h}^{i*}, \hat{x}_{2,l}^{i*} \) yield a higher value of the objective function than \( x_{2,h}^{i*}, x_{2,l}^{i*} \), which then cannot be a solution.

Proof of Lemma 3

Since we know \( c_{2,l}^{i} = 0 \), the relevant first order conditions are (84) and (86), together with the complementary slackness conditions (87) and (88). It is easy to see that both the participation and the limited commitment constraint cannot be slack: if this was the case, the lender could increase its revenues just by decreasing \( c_{2,h}^{i} \).

Claim 25 When \( \omega > \frac{1-\lambda i}{\alpha} p \), the participation constraint binds.

Proof. Suppose by contradiction the participation is slack:

\[
\alpha c_{1}^{i} + p c_{2,h}^{i} > \alpha \omega
\]
Then, the limited commitment constraint should bind: \( c_{2,h}^i = (1 - \lambda^i)\theta \). This gives that it must be \( c_1^i > 0 \). Indeed, if \( c_1^i = 0 \), then \( \alpha c_1^i + pc_{2,h}^i = p(1 - \lambda^i)\theta < \alpha\omega \). But then, since \( c_1^i > 0 \) and the participation constraint is slack, the lender can reduce \( c_1^i \), still satisfying all constraints, and increasing his expected utility, which is a contradiction.  

Define \( \lambda^* \) as the unique value of \( \lambda \) satisfying:

\[
\frac{\alpha - p}{1 - p} = \frac{u' \left( \frac{(1-\lambda)\theta\alpha}{\alpha} \right)}{u' \left( \theta + (1 - \lambda^*)\frac{\theta\alpha}{\alpha} (1 - \frac{\alpha}{\theta}) \right)}
\]  

(89)

Uniqueness is easy, because if we define the function

\[
F(\lambda) = \frac{u' \left( \frac{(1-\lambda)\theta\alpha}{\alpha} \right)}{u' \left( \theta + (1 - \lambda^*)\frac{\theta\alpha}{\alpha} (1 - \frac{\alpha}{\theta}) \right)}
\]

easily this function is increasing, \( F(1) > 0 \) by assumption and \( F(\lambda^*) = \frac{u' \left( \frac{(1-\lambda^*)\theta\alpha}{\alpha} \right)}{u' \left( \theta + (1 - \lambda^*)\frac{\theta\alpha}{\alpha} (1 - \frac{\alpha}{\theta}) \right)} \).

**Claim 26** For \( \omega > \frac{(1 - \lambda^i)\theta\alpha}{\alpha} \),

i If \( \lambda^i < \lambda^* \), both the participation constraint and the limited commitment constraints bind.

ii If \( \lambda^i \geq \lambda^* \), then the limited commitment constraint is slack.

\begin{align*}
&\text{a. } c_1 > 0 \text{ if } \omega \geq \frac{(1-\lambda^*)\theta\alpha}{\alpha} \\
&\text{b. } c_1 = 0 \text{ otherwise.}
\end{align*}

**Proof.** Suppose that both constraints are binding: then \( c_{2,h}^i = (1 - \lambda^i)\theta \) and \( c_1^i = \omega - p \left[ (1 - \lambda^i)\theta \right] \frac{\alpha}{\theta} \). From (86) we have

\[
\mu = \left[ \frac{p \left[ (1 - \lambda^i)\theta \right] \frac{\alpha}{\theta}}{\alpha} \right] > 0
\]

which replaced in (84) gives

\[
\eta = \frac{p}{\alpha} \left[ (\alpha - p)u' \left( \lambda^i\theta + \frac{(1 - \lambda^i)\theta\alpha}{\alpha} \right) - (1 - p)u' \left( \frac{(1 - \lambda^i)\theta\alpha}{\alpha} \right) \right]
\]

The consumption of the lender is

\[
x_{2,h}^i = \lambda^i\theta + \frac{(1 - \lambda^i)\theta\alpha}{\alpha}
\]

\[
x_{2,t}^i = \frac{(1 - \lambda^i)\theta\alpha}{\alpha}
\]

64
For this to be a solution, we need to check $c_1^i \geq 0$ and $\eta \geq 0$. This is the solution if and only if $
abla \omega > \frac{\begin{pmatrix} 1 - \lambda^i \end{pmatrix} p \theta}{\alpha}$ and

$$\frac{\alpha - p}{1 - p} > \frac{u'(\frac{\begin{pmatrix} 1 - \lambda^i \end{pmatrix} p \theta}{\alpha})}{u'(\lambda \theta + \frac{\begin{pmatrix} 1 - \lambda^i \end{pmatrix} p \theta}{\alpha})}$$

Next suppose that the limited commitment constraint is slack and the participation constraint is binding: then from (84)

$$\mu = u'(\omega - c_1^i + \theta - c_2^i) > 0$$

that replaced in (86) gives

$$p\mu + (1 - p)u'(\omega - c_1^i) \geq \alpha u'(\omega - c_1^i + \theta - c_2^i)$$

(90)

with equality if $c_1^i > 0$.

If $c_1 = 0$ then from (87) we obtain $c_2^i = \frac{\alpha \omega}{p}$ and the consumption of the lender is

$$x_{2,h}^i = \omega + \theta - \frac{\alpha \omega}{p}$$

$$x_{2,l}^i = \omega$$

We have left to check that the limited commitment constraint is slack, and that (90) is satisfied:

$$\omega \geq \frac{(1 - \lambda^i) p \theta}{\alpha}$$

(91)

$$\frac{u'(\omega)}{u'(\omega + \theta - \frac{\alpha \omega}{p})} > \frac{(\alpha - p)}{(1 - p)}$$

(92)

If instead $c_1 > 0$, from (87) we have $c_2^i = \frac{\alpha - c_1}{p}$. Lender’s consumption is

$$x_{2,h}^i = \omega - c_1 + \theta - \frac{\alpha (\omega - c_1)}{p}$$

$$x_{2,l}^i = \omega - c_1$$

where (90) holds at equality:

$$\frac{\alpha - p}{1 - p} = \frac{u'(\omega - c_1)}{u'(\omega + \theta - \frac{\alpha \omega}{p} + c_1(\frac{2}{p} - 1))}$$

(93)

We have left to check that the limited liability constraint is slack, which is the case if $c_{2,h}^i \geq (1 - \lambda^i)\theta \iff \omega \geq \frac{(1 - \lambda^i) p \theta}{\alpha}$.  

65
Therefore necessary and sufficient conditions for $c_1 = 0$ are $\omega \geq \frac{(1-\lambda^*) p\theta}{\alpha}$ and (92), whereas for $c_1 > 0$ are $\omega \geq \frac{(1-\lambda^*) p\theta}{\alpha}$ and (93). Combining (93) with the definition of $\lambda^*$ (89) it follows that $\omega - c_1 = (1 - \lambda^*) \frac{p\theta}{\alpha}$:

$$\frac{\alpha - p}{1 - p} = \frac{u'(\frac{(1-\lambda^*) p\theta}{\alpha})}{u'(\theta + (1 - \lambda^*) \frac{p\theta}{\alpha} (1 - \frac{\alpha}{p}))}$$

(94)

$$= \frac{u'(\omega - c_1)}{u'(\omega + \theta - \frac{\alpha \omega}{p} + c_1(\frac{\alpha}{p} - 1))}$$

(95)

$$\leq \frac{u'(\omega)}{u'(\theta + \omega(1 - \frac{\alpha}{p}))}$$

(96)

Therefore, given strict concavity of $u$, if $\omega < (1 - \lambda^*) \frac{p\theta}{\alpha}$ then $c_1 = 0$, and if $\omega \geq (1 - \lambda^*) \frac{p\theta}{\alpha}$ then $c_1 > 0$. ■

References


